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Liquidity and Capital Requirements and the Probability of Bank Failure

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Liquidity and Capital Requirements and the Probability of Bank Failure

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Abstract

Using the model of Rochet and Vives (2004), this note shows that a prudential regulator can in general not mitigate a bank's failure risk solely by means of liquidity requirements. However, their effectiveness can be restored if, in addition, minimum capital requirements are met. This provides a rationale for capital requirements beyond the commonly evoked reasoning that they are to be used to control the riskiness of banks' asset portfolios.

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1 Introduction

Can a prudential regulator reduce the risk of a bank failure *ex ante* by requiring the bank to hold a larger liquidity buffer?

The answer to this question is not clear-cut. On the one hand, endowing banks with a higher liquidity buffer enables them to withstand larger liquidity shocks without engaging in loss-generating fire-sales and thus reduces the risk of illiquidity. As the overall probability of bank failure is a function of the probabilities of illiquidity and of insolvency, the reduction in illiquidity risk also lowers the overall failure risk. Call this the *liquidity effect*. On the other hand, the liquidity effect is accompanied by a second effect which increases the probability of insolvency, thereby increasing the overall likelihood of failure. Call this the *solvency effect*. It arises because liquid assets earn lower returns on average and thus fail to generate the positive net returns which are needed to cover the bank's liabilities. In order to remain solvent, a larger liquidity buffer requires from the bank to achieve a higher minimal return out of its risky, productive investments and therefore the bank's insolvency risk rises with increases in liquidity holdings. Hence, there are two polar effects at work and it is *a priori* not clear which of the two outweighs the other. This in turn entails the danger that regulatory liquidity standards might actually raise, rather than lower, the probability of a banking failure.

However, by applying the bank-run model of Rochet and Vives (2004), this note shows that a prudential regulator can in fact reduce the bank's overall failure risk by means of liquidity requirements if and only if these are implemented in conjunction with minimum capital requirements. This result provides a rationale for minimum capital standards beyond the commonly evoked reasoning that they are to be used to control the riskiness of banks' asset portfolios.

In their original analysis, Rochet and Vives (2004) only focus on the liquidity effect and confirm that the liquidity effect has indeed a mitigating impact on the bank's failure risk if insolvency risk is not affected. But they neglect that, because of the different return characteristics of liquid and illiquid assets, the bank's insolvency risk is a function of its balance sheet composition. As liquidity requirements have a direct impact on the balance sheet composition, they directly affect the insolvency risk and put the solvency effect into action. In what follows, I solve Rochet and Vives's model by taking this dependency into account. The solution then implies that to become effective, liquidity buffers need to be backed by minimum capital requirements.

The remainder of the paper is organized as follows. Section 2 provides a condensed description of Rochet and Vives's model, which takes the dependency of insolvency on the balance sheet into account.² Section 3 contains the relevant comparative statics exercises. Section 4 discusses the main policy implication and concludes.

²For the complete derivation and discussion, the interested reader is referred to Rochet and Vives (2004).

2 Model

2.1 Set-Up

Rochet and Vives (2004) study a bank that operates at three dates, indexed by $\tau \in \{0, 1, 2\}$. At $\tau = 0$, the bank possesses equity of amount E_0 and takes in wholesale deposits $D_0 = 1$. The deposit contract promises a repayment $D > D_0$ independent of whether the funds are withdrawn at $\tau = 1$ or $\tau = 2$.

Deposits are managed by fund managers who decide on behalf of the original investors at $\tau = 1$ about rolling over or withdrawing the funds. In case they succeed in obtaining D , fund managers receive a fixed remuneration B . However, when they withdraw early at $\tau = 1$, they incur an additional cost of C . Limited liability implies that they receive a payoff of zero in case the bank fails.³ Fund managers then adopt the following behavioral rule: Withdraw at $\tau = 1$ if and only if

$$(B - C) - (1 - P)B > 0 \Leftrightarrow P > \gamma$$

where P is the probability that the bank fails and $\gamma := C/B$.

Of its funds, the bank invests amount I into a risky asset and keeps amount M in cash. Hence, its initial balance sheet at $\tau = 0$ reads

$$I + M = D_0 + E_0. \quad (1)$$

The return on the risky asset is a normally distributed⁴ random variable \tilde{R} with mean \bar{R} and precision (inverse variance) α . The risky asset is illiquid. It pays out the full return at $\tau = 2$, but can be sold at $\tau = 1$ only against a fire-sale price $R/(1 + \lambda)$, where R is the realization of the random return. Henceforth, λ is referred to as the fire-sale discount rate. Of course, cash is highly liquid in the sense that it can be freely used to cover liabilities at both $\tau = 1$ or $\tau = 2$.

2.2 Failure Conditions

Let x denote the proportion of deposits withdrawn early and let $m := M/D$ denote the liquidity ratio; the higher m is, the larger the interim deposit outflows can become which the bank can meet without the need to fire-sell assets. Thus, as long as $x < m$, the bank cannot come under liquidity stress at $\tau = 1$ and thus can only fail due to *insolvency* at $\tau = 2$. Precisely, it is insolvent whenever

$$R < R_s := \frac{D - M}{I}, \quad (2)$$

³Rochet and Vives (2004, p.1121) provide further discussion about the assumption that fund managers rather than depositors themselves manage the deposits.

⁴The assumption of normality is without loss of generality and is made for the purpose of obtaining tractable analytical solutions.

where R_s is called the *solvency threshold*. The initial balance sheet constraint (1) always holds and it can be used to express the solvency threshold as a function of the liquidity ratio m ,

$$R_s(m) = \frac{1 - m}{e - m}, \quad (3)$$

where $e := \frac{D_0 + E_0}{D}$. Whenever $e \geq 1$, the bank would be able to repay its liabilities without any risk. It is reasonable to assume $e < 1$, as Rochet and Vives implicitly do, which entails necessity of investment and thus $m < e$.

Absent further interim sources of liquidity, the bank has to resort to the market whenever $x > m$. In that case, it incurs liquidity costs per unit of the risky asset it liquidates. Total liquidity costs rise with x and even when the bank is able to cover total interim withdrawals xD , the associated liquidity costs can deplete its resources to such an extent that it is unable to meet the remaining liabilities $(1 - x)D$ at $\tau = 2$. Hence, the larger x is, the larger interim liquidity costs are, and therefore the higher the critical return must be above which the bank survives. The bank fails due to *illiquidity* at $\tau = 2$ if⁵

$$R < R_F(x, m) := R_s(m) \left(1 + \frac{\lambda \{x - m\}_+}{1 - m} \right). \quad (4)$$

$R_F(x, m)$ is (weakly) increasing in x . The operator $\{z\}_+$ abbreviates $\max\{0, z\}$.

Finally, the bank can already have failed at date 1 if liquidity pressure xD exceeded its total available resources $M + RI/(1 + \lambda)$. This entails early closure of the bank. The respective condition for R is given by

$$R < R_{EC}(x, m) := R_s(m)(1 + \lambda) \frac{\{x - m\}_+}{1 - m}. \quad (5)$$

Since early closure implies failure at date 2, $R_{EC}(x, m) \leq R_F(x, m)$.

The model described above is widely identical to Rochet and Vives's (2004) original model. The exception being that I have used the initial balance sheet constraint (1) in order to eliminate I from equation (2), which results in the solvency threshold R_s being a function of the liquidity ratio m , cf. equation (3). Rochet and Vives instead treat R_s as independent from m in their comparative statics, which implies that they switch off what I have called the solvency effect.⁶

2.3 Equilibrium

Observe that the bank always fails at $\tau = 2$ for any $R < R_s$ and it would always survive for any $R \geq (1 + \lambda)R_s := \lim_{x \rightarrow 1} R_F$. Under common knowledge of R , the model would exhibit

⁵For the explicit derivation consult Rochet and Vives (2004, p. 1124).

⁶Rochet and Vives (2004, p. 1124) use the balance sheet constraint to eliminate M from R_s and express R_s as a function of I . But then, on page 1125, they substitute out I by $(D - M)/R_s$ in the conditions for illiquidity failure while still maintaining that R_s is a function of I . Hence, their substitution entails a circularity and they do not correct this circularity in the derivation of the comparative statics result in their Proposition 3.

multiple Nash equilibria for $R \in [R_s, (1 + \lambda)R_s]$. Rochet and Vives therefore specify the model as an incomplete information game by assuming that fund managers only observe noisy signals about R . For a typical fund manager i ,

$$\tilde{s}_i = R + \varepsilon_i,$$

where ε_i is normally distributed around 0 with precision β . A strategy for a typical fund manager is then a mapping from the signal space \mathbb{R} into the set of actions {withdraw, roll over}. This structure permits the application of Global Game methods⁷ for the derivation of a unique equilibrium. I only state Rochet and Vives's result, for the proof, consult Rochet and Vives (2004, p.1128).

Proposition 1 (*Existence and Uniqueness of Equilibrium*)

When β is large enough relative to α , that is

$$\beta \geq \frac{1}{2\pi} \left(\frac{\lambda\alpha}{(e-m)} \right)^2 =: \beta_0,$$

there exists a unique Bayesian Nash Equilibrium where fund managers use symmetric switching strategies around t^* . A fund manager withdraws at date 1 whenever he observes $s_i < t^*$ and rolls over otherwise. The bank fails whenever $R < R^*$ and survives otherwise. The tuple (t^*, R^*) simultaneously solves the pair of equations

$$\Phi \left(\sqrt{\alpha + \beta}R - \frac{\alpha\bar{R} + \beta t}{\sqrt{\alpha + \beta}} \right) = \frac{C}{B} \quad (6)$$

and

$$R = R_s(m) \left(1 + \frac{\lambda\{\Phi(\sqrt{\beta}(t-R)) - m\}_+}{1-m} \right). \quad (7)$$

It follows that whenever $R_s < R^*$, an otherwise solvent bank can fail due to illiquidity. The ex ante risk of illiquidity is given by $\Pr[\tilde{R} \in (R_s, R^*)]$, while the ex ante insolvency risk is given by $\Pr[\tilde{R} < R_s]$. An illiquidity failure is brought about by a coordination failure: too many fund managers withdraw their deposits at $\tau = 1$, and although the bank would be perfectly able to meet all liabilities at $\tau = 2$, the illiquidity of its risky asset preclude it from doing so at $\tau = 1$ and it has to resort to the market to fire-sell assets to obtain liquidity. Absent any additional interim financing, the costs associated with these fire-sales are so large that the bank becomes unable to meet the remaining liabilities and goes bankrupt.

⁷See Morris and Shin (2003) for an extended survey of Global Games.

3 Comparative Statics

As already pointed out above, the difference between Rochet and Vives's original model and the version presented here is the explicit dependence of the solvency point on the balance sheet composition, reflected in the fact that R_s is a function of m . This entails that the effect of a change in m on R^* is composed of the liquidity effect *and* the solvency effect. Firstly, consider the latter. It affects R^* through the positive dependence of R_s on m . The derivative of R_s with respect to m is given by

$$\frac{\partial R_s(m)}{\partial m} = \frac{1 - e}{(e - m)^2} > 0. \quad (8)$$

Given its initial funding and given fixed liabilities $D > D_0 = 1$, any reduction in I (increase in M) implies that the bank must achieve a higher minimal return to remain solvent because cash holdings do not generate positive net returns which could be used to cover liabilities. This explains the positive sign of $\partial R_s(m)/\partial m$ in equation (8).

Secondly, consider the liquidity effect which works in the opposite direction. It becomes relevant only if $R^* > R_s$. For any given x , the larger m is, the smaller the exposure of the bank to adverse market condition becomes, implying that the depletion of resources through additional liquidity costs is less strong.

The following Lemma 1 provides the condition under which the liquidity effect outweighs the solvency effect. Intuitively, the liquidity effect dominates when the net return from holding a unit of the asset until $\tau = 1$ becomes negative. In that case, fire-sales generate less interim liquidity than cash holdings would have instead generated.

Lemma 1.

1. If $R^* = R_s$, the failure point R^* is strictly increasing in m .
2. If $R^* \in (R_s, (1 + \lambda)R_s)$, the failure point R^* is strictly decreasing (increasing) in m if and only if $R^* < (>)(1 + \lambda)$.

Proof. See Appendix.

4 Policy Implications

The previous section begs the question how a prudential regulator can exploit the liquidity effect in order to mitigate the illiquidity risk without stumbling across the solvency effect. Of course, the solvency effect can be minimized through larger investments into the risky asset or through larger equity holdings. The former would immediately raise the illiquidity risk, and would thus be counterproductive. However, the latter seems to be promising as it does not increase the liability burden and directly reduces the solvency threshold R_s . In order to clarify the latter point, suppose that the failure point is increasing in m . Then,

from Lemma 1 above, $R^* > (1 + \lambda)$. Further increases in m would only strengthen the condition. What can be done to reverse the inequality? An increase in e would do. As the failure point is strictly decreasing in e , it is possible to find a lower bound, denote it by \underline{e} , such that $R^* < (1 + \lambda)$ for all $e > \underline{e}$. As $e := (E_0 + D_0)/D$, the respective value \underline{E}_0 which sets $e = \underline{e}$ should then be implemented as the minimum capital requirement which would revitalize the effectiveness of liquidity buffers.

Lemma 2.

1. The failure point R^* is strictly decreasing in e .

2. $R^* < (1 + \lambda)$ if and only if

$$e \geq \underline{e} := \frac{1 + \lambda \Phi \left(\frac{\alpha}{\sqrt{\beta}}(1 + \lambda - \bar{R}) - \sqrt{\frac{\alpha + \beta}{\beta}} \Phi^{-1}(\gamma) \right)}{1 + \lambda}.$$

Thus, the minimum capital requirement which guarantees the effectiveness of liquidity buffers is given by

$$\underline{E}_0 = \max \left\{ 0, \frac{1 + \lambda \Phi \left(\frac{\alpha}{\sqrt{\beta}}(1 + \lambda - \bar{R}) - \sqrt{\frac{\alpha + \beta}{\beta}} \Phi^{-1}(\gamma) \right)}{1 + \lambda} D - 1 \right\} \quad (9)$$

Proof. See Appendix.

As the minimum capital requirement is a function of λ , it depends on the degree of financial stress in the economy. However, the sign of $d\underline{E}_0/d\lambda$ is not unambiguously clear in general. But for the limit cases of $\beta \rightarrow \infty$ (the case of infinitely precise signals), or $\alpha \rightarrow 0$ (a diffuse prior), \underline{E}_0 is strictly decreasing in λ . Hence, in these cases, the minimum capital requirement can be decreased in times of severe financial stress.

It should be emphasized that the previous Lemma does not provide a normative statement. It does not say anything about the desirability of using liquidity requirements as a tool of prudential regulation. Rather, depending on the respective objective function of the regulator, it is still possible that the respective amount of liquidity which minimizes the risk of bank failure becomes too costly to implement in terms of foregone returns (cf. Rochet and Vives (2004, p.)). However, the result allows to conclude that *if* prudential regulators want to use liquidity requirements as a control tool, then they would be well-advised not to implement them without paying attention to the bank's *capital-liability-ratio*. This provides a different rationale for minimum capital requirements. These do not only serve the purpose of controlling the riskiness of banks' asset portfolios, but also put the regulator in a position to effectively use other tools at its disposal to keep the different risks, which can lead to a banking failure, under control.

5 Appendix

Proof of Lemma 1.

1. The claim follows immediately from the derivative in equation (8).
2. Rewriting equation (6) gives

$$\sqrt{\beta}(t - R) = \frac{\alpha}{\sqrt{\beta}}(R - \bar{R}) - \sqrt{\frac{\alpha + \beta}{\beta}}\Phi^{-1}(\gamma).$$

Plugging the latter into equation (7) and using the definition of R_s from equation (3) yields

$$R = R_s(m) + \frac{\lambda \left\{ \frac{\alpha}{\sqrt{\beta}}(R - \bar{R}) - \sqrt{\frac{\alpha + \beta}{\beta}}\Phi^{-1}(\gamma) - m \right\}_+}{e - m}.$$

The latter implies that the equilibrium threshold R^* is given by the solution to

$$\psi(R, m) := (e - m)(R - R_s(m)) - \lambda \Phi \left(\frac{\alpha}{\sqrt{\beta}}(R - \bar{R}) - \sqrt{\frac{\alpha + \beta}{\beta}}\Phi^{-1}(\gamma) \right) + \lambda m = 0. \quad (10)$$

If $\beta > \beta_0$ (if equilibrium is unique), it follows that

$$\frac{\partial \psi(R, m)}{\partial R} = (e - m) - \lambda \varphi \left(\frac{\alpha}{\sqrt{\beta}}(R - \bar{R}) - \sqrt{\frac{\alpha + \beta}{\beta}}\Phi^{-1}(\gamma) \right) \frac{\alpha}{\sqrt{\beta}} > 0,$$

which allows the application of the implicit function theorem.

The failure point R^* can then be written as $R^* = r_\beta(m)$ with

$$r'_\beta(m) = -\frac{\partial \psi(R^*, m)/\partial m}{\partial \psi(R^*, m)/\partial R},$$

which is positive (negative) if

$$\frac{\partial \psi(R^*, m)}{\partial m} = -(R^* - R_s(m)) - (e - m) \frac{\partial R_s}{\partial m} + \lambda,$$

is negative (positive). Rewriting the latter, by using the definitions of $R_s(m)$ and $\partial R_s(m)/\partial m$, yields

$$\frac{\partial \psi(R^*, m)}{\partial m} \leq 0 \Leftrightarrow 1 + \lambda \leq R^*,$$

which proves the claim. \square .

Proof of Lemma 2.

1. From the proof of Lemma 1, $\partial\psi/\partial R > 0$. With

$$\frac{\partial\psi}{\partial e} = R > 0$$

follows

$$\frac{\partial R^*}{\partial e} = -\frac{\partial\psi/\partial e}{\partial\psi/\partial R} < 0,$$

as claimed.

2. Evaluating equation (10) at $R = (1 + \lambda)$ and using the definition of $R_s(m)$ gives

$$\psi(1+\lambda, m, e) = (e-m)(1+\lambda) - 1 + m - \lambda\Phi\left(\frac{\alpha}{\sqrt{\beta}}(1+\lambda - \bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right) + \lambda m = 0,$$

from which \underline{e} follows as

$$\underline{e} = \frac{1 + \lambda\Phi\left(\frac{\alpha}{\sqrt{\beta}}(1+\lambda - \bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right)}{1 + \lambda}$$

As R^* is strictly decreasing in e , any $e \geq \underline{e}$ guarantees $R^* < (1 + \lambda)$.

Finally, the minimum capital requirement can be calculated from the definitions of e and \underline{e} , and the inequality $e \geq \underline{e}$, by taking into account that initial equity cannot become negative. This proves the claim. \square

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