

# Flexibility of Wage Contracts and Monetary Policy\*

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## Abstract

The paper shows how indexation of wage contracts to the price level and the flexibility with which wages react to supply shocks depend on central bank preferences and the structure of uncertainty. The Barro–Gordon–model with endogenous and decentralized wage contracts is extended by explicit consideration of wage reactions to unanticipated productivity changes, by allowing wage contractors to pursue the goal of real wage stability, and by introducing sector specific shocks. It is shown under which conditions a rise in central bank independence, an increase in the variance of asymmetric shocks or the permission of indexed wage contracts lead to rising wage flexibility.

**Keywords:** monetary policy, monetary union, Phillips–curve, wage flexibility, wage indexation

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# I. INTRODUCTION

There is an ongoing debate about how central bank independence and monetary union influence the flexibility of wage contracts and the slope of the short-run Phillips-curve. This so called sacrifice ratio describes the increase in unemployment that is induced by a one percent decrease of inflation. Monetary policy can influence the unemployment rate via real wages. Its effectiveness depends crucially on the flexibility with which wages react to unexpected movements of prices and employment. The more indexation is built into wage contracts, the harder is it to drive a wedge between prices and wages, and the less effective is monetary policy. If wages react to supply shocks with sufficient flexibility, employment fluctuations can be avoided. Both, indexation and flexibility, reduce the incentive to use monetary policy as an instrument to stabilize employment. However, it may be in the interest of wage negotiators to maintain sticky wages. This paper presents a model to describe the effects of central bank preferences and asymmetric shocks on indexation and flexibility of wage contracts.

Ball (1988) assumed that private indexation decisions are aimed at minimizing employment fluctuations. He showed that equilibrium indexation is socially efficient if it is costless and the feedback on monetary policy is neglected. The interdependence of indexation decisions and discretionary monetary policy in the spirit of Barro and Gordon (1983) has first been analyzed by Devereux (1987). VanHoose and Waller (1991), Waller and vanHoose (1992), Milesi-Ferretti (1994) and Walsh (1995) showed that decentralized indexation creates a positive externality and may be inefficiently low. Their results on the equilibrium degree of indexation depend very sensible on the timing of decisions and random events. The same holds for the impact of uncertainty and central bank preferences. While in the models of VanHoose and Waller (1991) and Waller and VanHoose (1992) central bank preferences do not influence equilibrium indexation, Milesi-Ferretti (1994) finds that the optimal degree of indexation is the smaller, the bigger aggregate shocks are, the more weight is put on the goal of price stability and/or the smaller the output goal is. Walsh (1995) considered supply shocks that are observable by the central bank and demand shocks that are not. He showed that here, an increase in the weight placed on inflation may lower the equilibrium degree of indexation. Unfortunately,

his model may have three equilibria and the always existing one has full indexation independent of this weight.

In all of these papers it is assumed that wage negotiators minimize employment fluctuations. They could simply do so by allowing wages to react sufficiently strong on deviations of employment from its expected value. But, it is assumed that indexation to the aggregate price level is the only parameter of wage contracts. Flexibility in the sense of wage reactions to employment fluctuations is assumed to be zero, and it is not explained why wage contractors use an unsuitable instrument instead of a well-aimed one.

Bewley (1995, 1998) argued that employers are interested in wage stability as motivation for their workers' allegiance and to avoid bad morale. Fehr and Gächter (1998) attribute firms' reluctance to cut wages in a recession to reciprocity as a basic component of human behavior. For unions this objective follows from their members' risk aversion and from the insider-outsider theory. This gives rise to include a second objective for wage negotiations: Real wage stability should stand side by side with employment stability. This has first been suggested by Ball (1988). Ball and Cecchetti (1991) assumed a social loss function that depends on real wages only. They showed that for overlapping wage contracts inflation and welfare increase with the proportion of indexed contracts.

In this paper we use a Barro-Gordon model with decentralized wage bargaining. We assume that a wage contract has two components: a degree of indexation that ties nominal wages to the aggregate price level and a degree of flexibility with which wages may react to sector specific employment or productivity. These parameters are set by wage negotiators in order to minimize a weighted average of fluctuations in employment and real wages. We consider shocks that are observable by the central bank as well as shocks that it cannot observe. The latter may differ between economic sectors.

It will be shown that there is an interior optimum for the degree of flexibility that depends negative on the weight that contractors put on stabilizing real wages. The optimal degree of indexation is one. These results do neither depend on monetary policy nor on the magnitudes of the different kinds of shocks. But, it may be that the degrees of flexibility and indexation are constrained by legal or procedural

confinements. When this is taken into account, the constrained optima depend on monetary policy. We solve the model and show that there may be five types of equilibria with binding constraints.

The most interesting cases occur when maximal indexation is the only binding constraint. Here, a rise in the weight that the central bank puts on price stability (independence), a rising importance of asymmetric shocks (monetary union), a rise in the maximal degree of indexation and/or a drop in wage contractors concern for real wage stability leads to a rising degree of flexibility. The rationale: Independence and monetary union increase the need for flexible contracts. With rising independence the monetary authority is less *willing* to direct its policy towards employment stabilization, with rising asymmetric shocks it is less *able* to pursue this goal. Private decisions for flexible contracts can partially substitute for both of these effects. If higher indexation becomes possible, more flexible wage contracts will be accepted, because they reduce employment fluctuations, while real wage stability can be controlled for by higher indexation. This may be of special relevance for some European countries (e.g. Germany) where indexed contracts are illicit or socially banned.

If maximal flexibility is the only binding constraint, there may be multiple equilibria and the marginal effects of central bank preferences may be ambiguous. These equilibria do not depend on asymmetric shocks. It is important to note that the models of VanHoose and Waller (1991), Waller and VanHoose (1992) and Walsh (1995) are equivalent to special cases of this kind. One regard in which they are special is the neglect of asymmetric shocks. Our model shows that in their set-ups an inclusion of asymmetric shocks would not alter their results.

In the case that corresponds to Walsh (1995) there may be up to three equilibria with different comparative statics properties. Walsh's purpose, to show that wage indexation falls with rising central bank independence, can be served unambiguously if one considers only symmetric supply shocks that are observable by the central bank. In addition one has to assume a sufficiently large concern of wage negotiators for real wage stability.

While Cukierman and Lippi (1999), Grüner and Hefeker (1999), and Guzzo and Velasco (1999) aim at explaining the impacts of central bank independence and centralization of wage bargaining on the level of unemployment in non-stochastic

economies, this paper concentrates on fluctuations in a stochastic economy. The main difference in modelling is that they assume wage setters favouring high wages, while we proceed on the assumption that wage setters dislike wage fluctuations.

The next section introduces the basic assumptions of the model. Section III examines the optimization problem of wage setters and shows that within our framework flexible reactions of wages to employment and productivity are equivalent. Section IV analyzes the equilibrium when restrictions on flexibility and indexation are not binding. Section V derives the money supply rule for discretionary monetary policy. This allows to calculate the equilibria with binding constraints in section VI. Here, some results on comparative statics are given as well. Section VII deals with special cases and shows the relation to some other theories. Concluding remarks are given in section VIII.

## II. ASSUMPTIONS

VanHoose and Waller (1991) emphasized the role of timing of policy decisions and information use. They considered four cases differing in the information about shocks available to wage setters and monetary authority. In our model we represent this information by random variables realizing before and after the central bank's decision. The **time structure** of our model is as follows:

1. Wage setters in each sector agree upon a wage contract that specifies the nominal wage in their sector as a function of unexpected changes in the aggregate price level and of the sector specific economic situation.
2. An aggregate supply shock  $x$  realizes.<sup>1</sup>
3. Monetary authority sets money supply.
4. Another aggregate supply shock  $u$ , a demand shock  $v$ , and sectoral shocks  $\delta_i$  realize.
5. Firms decide on employment and production. Equilibrium prices are formed simultaneously. Wages are adjusted according to the agreed-upon contract.

The basic variables of our model are sectoral and aggregate nominal wages,  $w_i$  and  $w$ , the price level  $p$ , and sectoral and aggregate employment and output,  $l_i$ ,

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<sup>1</sup>It is well known that demand shocks that can be observed by the central bank will be fully compensated by monetary policy. Hence, we do not need to consider demand shocks at this stage.

$l$ ,  $y_i$ , and  $y$  respectively. Sectors are assumed to be small and indexed by  $i \in [0, 1]$ . Aggregate variables are defined by integrating their sectoral components, e.g.  $(w, l, y) = \int_0^1 (w_i, l_i, y_i) di$ . Small letters always denote logarithmic terms.

A **wage contract** for sector  $i$  is supposed to be a function

$$w_i := p^e + \gamma_i (p - p^e) + \varphi_i (l_i - l_i^e). \quad (1)$$

$(p^e, l_i^e) := E(p, l_i)$  are the expected price level and employment. Here, we assume rational expectations.  $\gamma_i$  is the degree of wage indexation to aggregate prices, and  $\varphi_i$  is a flexibility parameter that allows wage adjustments to unanticipated changes in employment. An alternative contract form would allow direct reactions of wages to supply shocks instead of employment. As we shall see in section III below, both contract forms are equivalent.

At stage 1 of our time structure wage setters specify contract parameters  $\gamma_i$  and  $\varphi_i$ . We assume that these parameters are set to minimize a weighted average of expected fluctuations in employment and real wages.

$$(\gamma_i, \varphi_i) := \arg \min_{\gamma_i, \varphi_i} \{ \text{Var}(l_i) + \zeta \text{Var}(w_i - p) \mid \gamma_i \in [0, \hat{\gamma}] \wedge \varphi_i \in [0, \hat{\varphi}] \}. \quad (2)$$

$\zeta \in [0, \infty]$  represents the relative weight given to the goal of stabilizing real wages. Employees and their representatives are interested in stabilizing employment and real wages because of risk aversion. Workers fear the threat of unemployment and want to plan their consumption expenses. Employers want to avoid employment fluctuations because of hiring and firing costs, especially a loss of firm specific human capital. Recent studies of Bewley (1995, 1998) show that firms are also interested in stable wages for keeping social peace and motivation of workers.

$\hat{\gamma} \in [0, 1]$  and  $\hat{\varphi} \in [0, \infty]$  restrict the maximal degrees of indexation and flexibility. They may stem from legal and social confinements as well as procedural limitations especially to the speed with which indisputable measures of prices and employment can be gained and used for wage adjustments.

**Discretionary monetary policy:** We assume that monetary authorities set money supply  $m$  in order to minimize a weighted average of price and employment fluctuations around desired levels. The desired level of employment  $l^*$  may be higher than

$E(l)$  because of labor market distortions (e.g. taxes and strategic behavior). The desired price level takes into account costs of expected and unexpected inflation. Menu costs are minimized when inflation is zero, while other costs stem from deviations of the price level from its expected value. This leads to a threefold objective function of the central bank.

$$m := \arg \min_m E \left( (l - l^*)^2 + \beta_1 p^2 + \beta_2 (p - p^e)^2 \mid x \right), \quad (3)$$

where  $\beta_1 \in [0, \infty]$  and  $\beta_2 \in [0, \infty]$  are the relative weights on the goals of price stability. This functional form is special and it must be said that quantitative results depend on it crucially. Nevertheless, it is the one that is used all over the literature, and qualitative results are robust against minor changes of the underlying preference relation.

We assume a continuum of **firms**  $i \in [0, 1]$ . The production function of firm  $i$  is given (in logarithmic terms) by

$$y_i := a l_i + \theta_i, \quad 0 < a < 1, \quad (4)$$

where  $\theta_i := x + u + \delta_i$  is the productivity shock faced by firm  $i$ .

Firms decide on labor demand and output by maximizing their profits. Hence, labor demand is given by

$$l_i := \ln \arg \max_{L_i} \{ P Y_i - W_i L_i \mid Y_i = \Theta_i L_i^a \}, \quad (5)$$

where capital letters denote the according non-logarithmic terms. Firms produce a homogeneous good and stand in perfect competition. Aggregate supply is given by  $y := \int_0^1 y_i d_i$ .

**Aggregate demand** is generated by the quantity equation, and prices are assumed to clear the goods market. This defines the price level as

$$p := m + v - y, \quad (6)$$

where  $v$  is a money demand shock.

We assume that the logarithmic **random terms**  $x$ ,  $u$ ,  $\delta_i$ , and  $v$  are pairwise independent of each other and distributed with variances  $\sigma_x^2$ ,  $\sigma_u^2$ ,  $\sigma_i^2$ , and  $\sigma_v^2$  around means of zero.  $\delta_i$  should represent the deviations of sector  $i$ 's productivity from the aggregate shock  $x + u$ . Hence, we can assume without loss of generality that  $\int \delta_i di \equiv 0$  or  $\theta \equiv x + u$ .

**Short run labor supply** is assumed to be high enough to meet whatever is demanded, so there is no rationing of firms even in the case of positive shocks. We shall calculate the expected values under this hypothesis. Consistency requires that the support of our random terms is limited above, because otherwise short-run labor supply would have to be infinite.

### III. WAGE SETTERS' REACTION FUNCTIONS

Let us start our analysis by considering the firms' decisions. Note, that we identified a sector, for which a wage contract is settled, with a profit maximizing firm. For  $\varphi_i \neq 0$ , this has the effect that firm  $i$  can influence the wage it has to pay. We take this into account, but, we should emphasize that it may be perfectly reasonable to assume that there is a continuum of firms in each wage negotiating sector, so that wages are beyond a single firm's reach. Maximizing profits as described by (5) with respect to (1) yields

$$l_i = \bar{l} + \frac{1}{1-a} [p - w_i + \theta_i - \rho_i \ln(1 + \varphi_i)], \quad (7)$$

where  $\bar{l} := \ln a / (1 - a)$ . The last term is due to the influence of firms on wages. The parameter  $\rho_i \in [0, 1]$  stands for the strength of this effect.  $\rho_i$  should be one if sector  $i$  consists of a single firm. It should be zero if the wage contract holds for a large number of relatively small firms.

From (1) and (7) we get

$$w_i = p^e + (1 - \lambda_i) (p - p^e) + \phi_i \theta_i, \quad (8)$$

where  $\lambda_i := \frac{(1-a)(1-\gamma_i)}{1-a+\varphi_i}$  and  $\phi_i := \frac{\varphi_i}{1-a+\varphi_i}$ . This shows that contract form (1) is equivalent to a contract form, where wages react to supply disturbances  $\theta_i$  directly.



$\lambda_i$  can be interpreted as the degree to which wages stay behind the price level if it exceeds its expected value.  $\phi_i$  may be viewed at as the speed of wage adjustments to supply shocks. Since  $\theta_i$  is a measure of productivity in sector  $i$ ,  $\phi_i$  may also be interpreted as a sector specific productivity bonus. The equivalence of (1) and (8) shows that within our framework a productivity bonus is a perfect substitute to flexible responses of wages to employment. This relates closely to Karni (1983) who showed that an optimal indexation of wages to aggregate prices and output can duplicate the equilibrium that would obtain if wages could be conditioned on the shocks directly. The common reason for both results is that indexation to variables whose fluctuations allow to detect the exact magnitude of underlying shocks can be replaced by direct indexation to these shocks.

The relation between  $(\gamma_i, \varphi_i)$  and  $(\lambda_i, \phi_i)$  is one-to-one. We shall proceed our analysis by using contract form (8), because it makes calculations easier. Please notice that limits of flexibility and indexation are now transformed into  $\hat{\phi} := \frac{\hat{\varphi}}{1-a+\hat{\varphi}} \in [0, 1]$  and a lower bound on  $\lambda_i$  given by  $\check{\lambda}_i := \frac{(1-a)(1-\hat{\gamma})}{1-a+\hat{\varphi}} \in [0, 1]$ .

Using these definitions and (2), (7), and (8), we yield the **reaction function of wage setters** in sector  $i$  as

$$(\lambda_i, \phi_i) = \arg \min_{\lambda_i, \phi_i} \left\{ f(\lambda_i, \phi_i) \mid \lambda_i \in [\check{\lambda}_i, 1] \wedge \phi_i \in [0, \hat{\phi}] \right\}, \quad (9)$$

where

$$f(\lambda_i, \phi_i) := \lambda_i^2 z \text{Var}(p) + 2 \lambda_i (1 - z \phi_i) \text{Cov}(p, \theta_i) + (z \phi_i - 2) \phi_i \text{Var}(\theta_i)$$

and  $z := 1 + \zeta(1 - a)^2$ . This gives us the individually optimal degrees of indexation and flexibility for given fluctuations of aggregate prices.

## IV. UNCONSTRAINED EQUILIBRIUM

Let us assume for a moment that the limits to flexibility and indexation are not binding, Then, the optimal values for  $\lambda_i$  and  $\phi_i$  are given by the first order conditions

$$f_\lambda/2 = z \lambda_i \text{Var}(p) + (1 - z \phi_i) \text{Cov}(p, \theta_i) = 0 \quad \Leftrightarrow \quad \phi_i = \frac{1}{z} + \frac{\text{Var}(p)}{\text{Cov}(p, \theta_i)} \lambda_i \quad (10)$$

and

$$f_\phi/2 = -z \lambda_i \text{Cov}(p, \theta_i) - (1 - z \phi_i) \text{Var}(\theta_i) = 0 \quad \Leftrightarrow \quad \phi_i = \frac{1}{z} + \frac{\text{Cov}(p, \theta_i)}{\text{Var}(\theta_i)} \lambda_i. \quad (11)$$

They imply

$$(\lambda_i, \phi_i) = (0, 1/z) \quad \vee \quad \text{Var}(p) \text{Var}(\theta_i) = (\text{Cov}(p, \theta_i))^2. \quad (12)$$

As will be shown more rigorously later on, the latter equation can hold only if three out of the four random terms are set equal to zero. In all other cases  $\phi = 1/z$  and  $\lambda = 0$  is the only equilibrium with non-binding constraints. So, there is an interior optimum for the wage reaction to supply shocks. More flexibility in wages would hurt the desired stability of real wages, less flexibility would lead to too strong fluctuations in employment. With direct reactions of wages to supply shocks, the remaining purpose of indexation is to cushion demand shocks. In the unconstrained equilibrium full indexation neutralizes these shocks.

Transformed into the parameters of our original contract form (1), the equilibrium is at  $\varphi = \frac{1}{\zeta(1-a)}$  and  $\gamma = 1$ . The resulting equilibrium deviations of real wages and employment from their expected values are

$$w_i - p = \frac{1}{z} \theta_i \quad \text{and} \quad l_i - \text{E}(l_i) = \left(1 - \frac{1}{z}\right) \frac{\theta_i}{1-a}. \quad (13)$$

If wage setters are only interested in stabilizing employment ( $z = 1$ ) we get the obvious result that  $\phi = 1$ . Supply shocks are fully compensated by real wages while demand shocks are neutralized by indexation, so that employment does not fluctuate at all. On the other hand, if wage setters are only concerned about real wages ( $z = \infty$ ) then  $\phi = 0$ , supply shocks lead to appropriate fluctuations in employment, and real wages are completely stabilized by indexation.

While the unrestricted equilibrium is independent of monetary policy, this is not the case if  $\hat{\phi} < 1/z$  or  $\check{\lambda}_i > 0$ .

## V. MONETARY POLICY

Indexation and flexibility of wages influence the slope of the Phillips–curve and the extent to which it is shifted by supply shocks. Therefore, these parameters have an important impact on monetary policy. The more inflation is necessary to create a given number of jobs, i.e. the lower is the sacrifice ratio, the less will a monetary authority direct its policy towards output stabilization. To see this relation we derive the optimal monetary policy for given wage contracts. From equations (7) and (8) we get the short–run Phillips–curve

$$l = \bar{l} + \frac{1}{1-a}[\lambda(p - p^e) + \theta - I], \quad (14)$$

where  $I := \int_0^1 (\phi_i \theta_i + \rho_i \ln(1 + \varphi_i)) di$ . Using (14), the aggregate production function  $y = al + \theta$  and quantity equation (6) imply

$$p = p^e + \frac{1}{1+c\lambda}[m + v - \bar{y} - p^e - (1+c)\theta + cI], \quad (15)$$

where  $\bar{y} := a\bar{l}$  and  $c := a/(1-a)$ .

Solving the first order condition of the central bank's optimization problem (3) with respect to (14) and (15) yields the money supply rule

$$m = \bar{y} + p^e + \frac{(1+c\lambda)(c\lambda k - b_1 p^e) + (b(1+c) - c^2\lambda(1-\lambda))x + (c\lambda - b)cE(I|x)}{c^2\lambda^2 + b} \quad (16)$$

where  $k := a(l^* - \bar{l})$ ,  $b_1 := a^2\beta_1$ , and  $b := a^2(\beta_1 + \beta_2)$ .

Using (15), rational expectations imply

$$E(m) = p^e + \bar{y} - cE(I). \quad (17)$$

From this and (16), we find

$$p^e = \frac{c\lambda}{b_1}[k + cE(I)]. \quad (18)$$

Inserting (18) into (16) allows to rewrite the monetary authority's reaction to supply shocks as

$$m = E(m) + (1 + \mu)x, \quad (19)$$

where

$$\mu := \frac{c(b - c\lambda)(1 - \phi)}{c^2\lambda^2 + b}.$$

$\mu$  is a measure of the central bank's response to supply shocks. It is positive iff  $b > c\lambda$  and  $\phi < 1$ . In this case money supply reacts overproportional to observable supply shocks.

With full indexation ( $\lambda = 0$ ), we get  $\mu = c(1 - \phi)$ . Here, money supply does not depend on the central bank's preferences, because it cannot influence employment. In this case the central bank will only pursue the goal of price stability; the inflation bias disappears and employment fluctuations can only be influenced by flexibility  $\phi$ . Note, that this happens in the unconstrained equilibrium.

With rising  $\lambda$  the sacrifice ratio rises and the central bank will put more effort into stabilizing employment.  $\mu$  becomes smaller and will be negative for  $\lambda > b/c$ . If  $\lambda$  rises above  $(b + \sqrt{b^2 + b})/c$  then  $\mu$  starts rising again, but remains negative. Note, that this can happen only if  $b \ll c$ . For  $b > c$ ,  $\mu$  is positive even if wages are not indexed at all. However, with this money supply rule, expected employment fluctuations monotonically increase with rising degree of indexation, as can be seen from

$$E(l - l^e | x) = \frac{\mu\lambda + 1 - \phi}{1 + c\lambda} \frac{x}{1 - a} = \frac{(1 - \phi)b}{c^2\lambda^2 + b} \frac{x}{1 - a}. \quad (20)$$

For  $0 < \lambda < 1/c$ ,  $\mu$  increases in  $b$ . The higher the weight the monetary authority attaches to the goal of price stability, the more will the money supply be aligned to supply shocks. This prevents prices to fall [rise] too much in case of positive [negative] shocks. This can better be seen from the impact of monetary policy on the price level.

$$E(p - p^e | x) = \frac{\mu - c(1 - \phi)}{1 + c\lambda} x = \frac{-c^2\lambda(1 - \phi)}{c^2\lambda^2 + b} x. \quad (21)$$

Since  $\mu \leq c(1 - \phi)$  for  $\phi < 1$ , prices always move in the opposite direction of productivity. Given the central bank's information, prices are expected to fluctuate the more, the smaller  $\mu$  is, while expected employment fluctuations rise in  $\mu$ .

$|\mu|$  falls with rising wage flexibility, because  $\mu$  is a partial substitute for  $\phi$  with respect to the goal that is more relevant for monetary policy. If the central bank directs its policy towards price stabilization, then  $\mu$  is positive and a partial substitute for wage flexibility in its effect on expected price fluctuations. If central bank policy is directed more towards to the output goal, then  $\mu$  is negative and a partial substitute for wage flexibility in its effect on employment.

If  $\phi = 1$  then  $\mu = 0$  and expected deviations of prices and employment are zero. Deviations can only occur because of shocks that are not yet observable by the central bank. But, there is still an inflation bias stemming from the central bank's desire to rise employment above its expected value. So, while full wage flexibility ( $\phi = 1$ ) enables the monetary authority to keep fluctuations in prices and employment at a minimum, average inflation can be brought to zero only with full indexation.

## VI. CONSTRAINED EQUILIBRIA

For simplicity we confine ourselves to symmetric equilibria. They only make sense when  $\sigma_i$  is the same for all sections. In a symmetric equilibrium  $I - E(I) = \phi(x + u)$ . Using this and equations (15) and (19) the aggregate price level is given by

$$p = p^e - \frac{c^2 \lambda (1 - \phi)}{c^2 \lambda^2 + b} x - \frac{1}{1 + c \lambda} [(1 + c(1 - \phi)) u - v]. \quad (22)$$

Using this and the properties of random terms we find

$$\text{Var}(p) = \left( \frac{c^2 \lambda (1 - \phi)}{c^2 \lambda^2 + b} \right)^2 \sigma_x^2 + \left( \frac{1 + c(1 - \phi)}{1 + c \lambda} \right)^2 \sigma_u^2 + \frac{1}{(1 + c \lambda)^2} \sigma_v^2, \quad (23)$$

$$\text{Cov}(p, \theta_i) = - \frac{c^2 \lambda (1 - \phi)}{c^2 \lambda^2 + b} \sigma_x^2 - \frac{1 + c(1 - \phi)}{1 + c \lambda} \sigma_u^2 \leq 0, \quad (24)$$

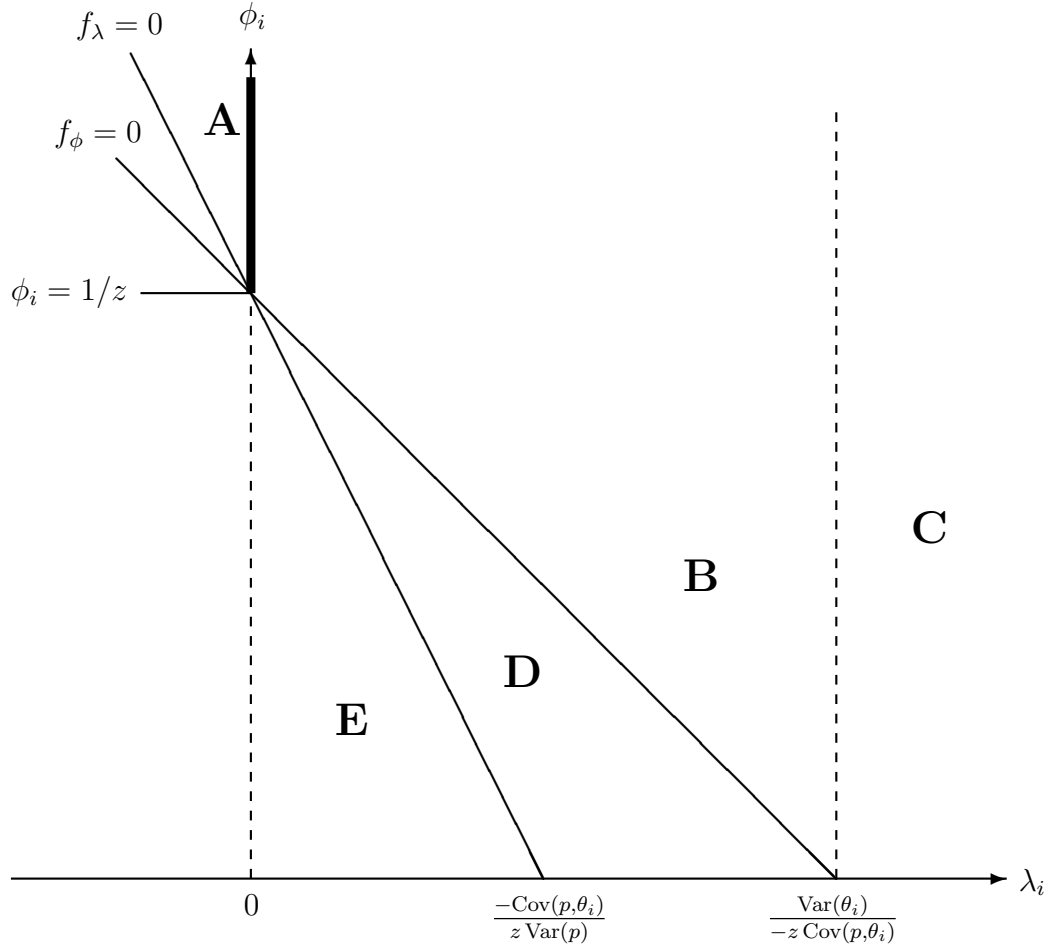
$$\text{Var}(\theta_i) = \sigma_x^2 + \sigma_u^2 + \sigma_i^2. \quad (25)$$

It is now easy to see that

$$\text{Var}(p) \text{Var}(\theta_i) \geq (\text{Cov}(p, \theta_i))^2. \quad (26)$$

(26) holds as equality if and only if three out of the four variances  $\sigma_x^2$ ,  $\sigma_u^2$ ,  $\sigma_i^2$ , and  $\sigma_v^2$  equal zero.

The optimal reaction of individual wage setters to given aggregates and restrictions can best be seen from Figure 1:



**Figure 1** The elliptical curves represent indifference curves of wage setters in sector  $i$ . Note, that the slopes of the lines for which  $f_\lambda = 0$  and  $f_\phi = 0$  depend on aggregate values of  $\lambda$  and  $\phi$ . The facts that both lines have a negative slope and that  $f_\lambda = 0$  is steeper than  $f_\phi = 0$  follow from (24) and (26).

There are six different cases to consider:

**A.** If  $\hat{\phi} \geq 1/z$  and  $\check{\lambda} = 0$  then wage setters will choose the unconstrained optimum as described in section IV above.

**B.** If  $0 < \check{\lambda} \leq \frac{\text{Var}(\theta_i)}{-z \text{Cov}(p, \theta_i)}$  and  $\hat{\phi} \geq \frac{1}{z} + \check{\lambda} \frac{\text{Cov}(p, \theta_i)}{\text{Var}(\theta_i)}$ , that is  $(\check{\lambda}, \hat{\phi})$  are in region B, then  $\lambda_i = \check{\lambda}$  and  $\phi_i = \frac{1}{z} + \check{\lambda} \frac{\text{Cov}(p, \theta_i)}{\text{Var}(\theta_i)}$ .

In a symmetric equilibrium of this type

$$\phi = 1 - \frac{\frac{\check{\lambda}}{1+c\check{\lambda}} \sigma_u^2 + \frac{z-1}{z} (\sigma_x^2 + \sigma_u^2 + \sigma_i^2)}{\frac{b}{c^2 \check{\lambda}^2 + b} \sigma_x^2 + \frac{1}{1+c\check{\lambda}} \sigma_u^2 + \sigma_i^2}. \quad (27)$$

$\phi$  rises in  $b$  and  $\sigma_i^2$ . It falls with rising  $\check{\lambda}$  or  $z$ . If the central bank puts more weight on price stability the impact of  $x$ -shocks is shifted from prices to employment. Since wages are not fully indexed in this case, greater price stability carries over to more stability in real wages. An increase in wage flexibility has an opposite effect on real wages and employment and will be used to rebalance both kinds of uncertainty according to the preferences of wage negotiators.

Rising variance of asymmetric shocks hits employment relatively harder than real wages, because shocks in singular sectors cannot affect the aggregate price level. An increase in flexibility shifts a part of this additional uncertainty over to real wages and brings the relation between real wage and employment fluctuations closer to the desired level.

A decrease in the relative weight that contractors put on real wage stability leads to higher flexibility, because that shifts the impact of productivity shocks from real wages to employment.

It is interesting to note the dependency of  $\phi$  on  $\check{\lambda}$  in this case. If restrictions on wage indexation are relaxed, wage flexibility increases. The reason is that wage indexation partially offsets the negative effect that a high degree of flexibility has on the variance of real wages. A relaxation of these restrictions can reduce employment fluctuations because it allows wage contractors to accept more flexibility without having to fear immoderate real wage fluctuations. The inflexibility of wages in some European countries may be partly due to interdictions of indexed wage contracts.

If the central bank puts more weight on price stability or if the relative importance of asymmetric shocks rises, wage contracts respond more flexible to supply shocks. This holds until eventually the maximal degree of flexibility is reached which means that the equilibrium type changes from B to D. Since  $\phi$  falls with rising  $\check{\lambda}$  or  $z$ , a relaxation of jurisdictional limitations to indexation or a decrease in the relative weight that contractors put on stabilizing real wages has the same effect.

**C.** If  $\check{\lambda} > \frac{\text{Var}(\theta_i)}{-z \text{Cov}(p, \theta_i)}$ , then  $(\check{\lambda}, \hat{\phi})$  are in region C. In this case  $\lambda_i = \check{\lambda}$  again and  $\phi_i = 0$ .

A symmetric equilibrium of this type exists iff  $\check{\lambda} > 0$  and

$$z \frac{c^2 \check{\lambda}^2}{c^2 \check{\lambda}^2 + b} \sigma_x^2 + z \frac{\check{\lambda} + c \check{\lambda}}{1 + c \check{\lambda}} \sigma_u^2 > \sigma_x^2 + \sigma_u^2 + \sigma_i^2 \quad (28)$$

A sufficient condition for this inequality to hold for all  $\sigma_x^2$  and  $\sigma_u^2$ , is

$$\beta_1 + \beta_2 < \zeta \check{\lambda}^2 \quad \wedge \quad \frac{\sigma_i^2}{\sigma_u^2} < z \frac{\check{\lambda} + c \check{\lambda}}{1 + c \check{\lambda}} - 1. \quad (29)$$

Thus, if indexation of wages is not allowed ( $\check{\lambda} = 1$ ), the central bank's weight on price stability is lower than the wage contractors' desire for real wage stability, and asymmetric shocks are sufficiently small, the conditions for an equilibrium of type C hold. They also hold when  $\check{\lambda} = 1$  and  $\sigma_x^2$  and  $\sigma_i^2$  are both relatively small compared to  $\sigma_u^2$ . The latter may describe the situation in some European economies.

The only marginal change that can influence the contract parameters in this case is a change in the maximal degree of indexation. But, since the left side of inequality (28) is falling with falling  $\check{\lambda}$ , falling  $z$ , and rising  $b$ , substantial parameter changes into this direction can drive the economy out of a type C-equilibrium. Those changes would tend to bring it to an equilibrium of type B with a resulting positive degree of wage flexibility. The same happens if the relative importance of asymmetric shocks increases substantially, as should be expected from a monetary union.

**D.** If  $\left(\frac{1}{z} - \hat{\phi}\right) \frac{-\text{Cov}(p, \theta_i)}{\text{Var}(p)} < \check{\lambda} < \left(\frac{1}{z} - \hat{\phi}\right) \frac{\text{Var}(\theta_i)}{-\text{Cov}(p, \theta_i)}$ , the restrictions are in region D,  $\lambda_i = \check{\lambda}$  and  $\phi_i = \hat{\phi}$ .



The restrictions on maximal indexation and flexibility are both binding. Marginal changes of  $\check{\lambda}$  and  $\hat{\phi}$  shift the equilibrium in the obvious directions. Changes of other exogenous variables influence contract parameters only if they are large enough to violate the conditions for a type D-equilibrium.

**E1.** If  $\hat{\phi} < 1/z$  and  $\check{\lambda} \leq \left(\frac{1}{z} - \hat{\phi}\right) \frac{-\text{Cov}(p, \theta_i)}{\text{Var}(p)} \leq 1$ , the restrictions are in region E and  $\phi_i = \hat{\phi}$  while  $\lambda_i = \left(\frac{1}{z} - \hat{\phi}\right) \frac{-\text{Cov}(p, \theta_i)}{\text{Var}(p)}$ .

A symmetric equilibrium of type E1 is a solution to  $\lambda = \lambda_i$  as above with respect to (23), (24), and  $\phi = \hat{\phi}$ . Since this is a polynomial of fifth degree, there may be multiple solutions. The marginal effects of changes in  $b$ ,  $z$ ,  $\hat{\phi}$ , and  $\sigma_v^2$  are ambiguous and will be analyzed below for special cases only.  $\sigma_i^2$  does not affect the equilibrium in this case.

**E2.** If  $\hat{\phi} < \frac{1}{z} + \frac{\text{Var}(p)}{\text{Cov}(p, \theta_i)}$ , the restrictions are in region E again and  $\phi_i = \hat{\phi}$ . But here, wage setters would prefer a negative indexation that is excluded by assumption. So, they will choose the maximal  $\lambda_i = 1$ .

A symmetric equilibrium of this type exists iff  $\hat{\phi} < 1/z$  and

$$\frac{(1 - \hat{\phi})c^2}{c^2 + b} \left[ \frac{1}{z} - \frac{c^2 + b\hat{\phi}}{c^2 + b} \right] \sigma_x^2 + \frac{1 + c(1 - \hat{\phi})}{1 + c} \left[ \frac{1}{z} - \frac{1 + c - \hat{\phi}}{1 + c} \right] \sigma_u^2 > \frac{1}{(1 + c)^2} \sigma_v^2. \quad (30)$$

Rising demand uncertainty can lead to violations of (30) and to equilibria of type E1 or D with positive indexation. A sufficient condition for (30) to hold for all  $\sigma_x^2$  and  $\sigma_u^2$  is

$$b > \frac{(z - 1)c^2}{1 - z\hat{\phi}} \quad \wedge \quad \frac{\sigma_v^2}{\sigma_u^2} < \hat{\phi} - \frac{z - 1}{z} (1 + c). \quad (31)$$

If  $\hat{\phi}$  is smaller than  $1/z$  but larger than  $(1+c)(z-1)/z$  and if central bank's preference for price stability is sufficiently large and demand shocks are neglectable then there exists an equilibrium of type E2.

## VII. SPECIAL CASES

One effect that can easily be seen from figure 1 is the influence of the weight that wage contractors put on the goal of real wage stabilization. An increase in this weight, i.e. an increase in  $z$ , shifts the lines where  $f_\lambda = 0$  and  $f_\phi = 0$  downward. This shifts regions B and D, increases A and C and reduces E. In the extreme case where contractors are only concerned about real wage fluctuations ( $z = \infty$ ), sufficient conditions for case C hold whenever  $\check{\lambda} > 0$ . If wage setters only care about employment ( $z = 1$ ), condition (28) is violated and the equilibrium cannot be of type C.

In VanHoose and Waller (1991), Waller and VanHoose (1992), and Walsh (1995)  $z = 1$  and wage contracts cannot respond to supply shocks directly, while there is no limit to indexation, i.e.  $\hat{\phi} = \check{\lambda} = 0$ . It is easy to see that this rules out cases A – D. Nor do they consider asymmetric shocks, but this is irrelevant, because type E-equilibria do not depend on  $\sigma_i^2$  anyway.

In addition, Walsh (1995) assumes that the central bank can observe all supply shocks. We can represent this by setting  $\sigma_u^2 = 0$ . Given this specialization, an equilibrium of type E1 is a solution of

$$\lambda(c^2 \lambda^2 + b) \sigma_v = \lambda c \sqrt{b}(1 + c \lambda) \sigma_x. \quad (32)$$

This is equivalent to Walsh's equation (10). Obviously, full indexation ( $\lambda = 0$ ) is such an equilibrium. If and only if

$$\frac{\sigma_v^2}{\sigma_x^2} < \frac{c^2}{4} \left( 1 + 2 \frac{1 + \sqrt{1 + b}}{b} \right) \quad (33)$$

equation (32) has two more solutions, say  $\lambda_1$  and  $\lambda_2$ . They are additional equilibria of type E1 provided that they are contained in  $(0, 1]$ .  $\lambda_1$  is positive iff  $\sigma_v^2/\sigma_x^2 > c^2/b$ . Then, it rises in  $b$ , so that an increase in the central bank's preference for price stability reduces indexation in this equilibrium. This effect has been pointed out by Walsh (1995).  $\lambda_2$  is smaller than one iff  $\frac{\sigma_v^2}{\sigma_x^2} > \max \left\{ \frac{b}{4}, \frac{bc^2(1+c)^2}{(b+c^2)^2} \right\}$ . It may rise or fall with  $b$ , depending on the parameter constellations. So, in this equilibrium the degree

of indexation may rise with rising preference for price stability. For  $\frac{\sigma_v^2}{\sigma_x^2} < b \left( \frac{c+c^2}{b+c^2} \right)^2$ , there is an equilibrium of type E2 with no indexation.

In Waller and VanHoose (1992) it is assumed that supply shocks cannot be observed by the central bank. We can reproduce their results by setting  $\sigma_x^2 = 0$ . Then, there is only one equilibrium of type E1 where the degree of indexation is

$$1 - \lambda = \frac{\sigma_v^2}{(1+c)\sigma_u^2 + \sigma_v^2}. \quad (34)$$

Another interesting case is the one where only symmetric supply shocks, observable by the central bank, are considered. This is represented by  $\sigma_u^2 = \sigma_v^2 = \sigma_i^2 = 0$ . Furthermore, let us assume that wages cannot react to unexpected deviations of employment or supply shocks directly, as has been assumed in the previous literature, and that full indexation is possible, i.e.  $\hat{\phi} = \check{\lambda} = 0$ . Now, the equilibrium degree of indexation is given by

$$\lambda = \min \left\{ 1, \sqrt{(\beta_1 + \beta_2)/\zeta} \right\}. \quad (35)$$

Obviously,  $\lambda$  rises in  $\beta_1 + \beta_2$  until the central banks' weight on price stability equals the privates' weight on real wage stability. This shows that the decrease in indexation in response to increasing central bank independence, as emphasized by Walsh (1995), can be arrived at without reference to demand shocks, if one assumes that wage negotiators have a sufficiently strong interest in real wage stability.

Finally, if  $\check{\lambda} = 1$ , i.e. indexation is not allowed, as is the case in Germany, then the equilibrium is of one of the types B, C, D, or E2, with E1 being the border case between C and E2. A marginal impact of central bank independence exists only in case B where flexibility unambiguously rises with  $b$  from zero to its maximal value.

## VIII. CONCLUDING REMARKS

We presented a model that explains the interaction between wage flexibility and monetary policy. It generalizes models found in the previous literature in two respects: First, we included the possibility that nominal wages may react to unexpected changes in employment or productivity. Second, we allowed for wage contractors to put a positive weight on the goal of real wage stability. Both extensions

stand on firm ground and are logically consistent with the basic ideas behind this line of research.

We have given a rather general description of different types of symmetric equilibria and shown the impact of changes in exogenous parameters on most of them. If neither the maximal degree of indexation nor maximal flexibility are binding constraints then wage contractors will choose full indexation and a degree of flexibility that rises with the relative weight that they put on their employment goal. Monetary policy does not influence this solution.

A marginal influence of central bank preferences on the equilibrium degree of flexibility exists only if maximal indexation is a binding constraint for wage negotiators. Flexibility of wage contracts tends to rise with rising independence of the central bank and with the relative importance of asymmetric shocks. This is a good case to describe the arguments brought forward in context with the European monetary union. Somewhat surprisingly, a relaxation of limits to indexation increases flexibility as well. Reason is that indexation and flexibility have opposite effects on real wage stability. If wages react more flexible to the labor market, real wage fluctuations increase. This is partially offset by a higher degree of indexation, because prices always move in opposite direction of supply shocks.

The analysis of some special cases has shown that they accord with results of the existing literature. We can reproduce these results if we refrain from modeling flexibility and just look on the impact of central bank preferences on indexation. We have also shown that, increasing central bank independence leads to a reduction in the equilibrium degree of indexation if the only shocks considered are supply shocks observable by the central bank.

We calculated the solutions to our model for wage contracts, in which flexibility takes the form of a productivity bonus. We argued that this is equivalent to a contract form, where wages react to employment. Using the coordinate transformation between  $(\lambda, \phi)$  and  $(\gamma, \varphi)$  as given in section III it is easy to see that figure 1, the qualitative results, and all rationales carry over to the other pair of contract parameters.

Our model emphasized *fluctuations* of employment, prices, and real wages. Strategic considerations of firms, as they occur when wages depend on employment, the

central bank's employment target, and the composition of costs of expected and unexpected inflation influence the *levels* of employment, wages, and inflation, but not their fluctuations. This explains why the respective parameters do not enter our equilibrium conditions.

While indexation is interdicted in some countries, it is rather difficult to give a good justification for a strict upper limit to flexibility of wage reactions to (un)employment or to productivity bonuses. It might be more convincing to think of *costs* associated with flexibility and indexation. If this cost function is smooth, and costs are zero for fixed nominal wages and finite for all positive degrees of indexation and flexibility, we would only get interior optima. However, this does not change our results as dramatically as it might seem at first glance. There will still be parameter regions with associated equilibria that behave like in our regions B, C, and E, depending on whether the costs of flexibility or the costs of indexation dominate. The advantage of our way of modeling is to emphasize these effects even if conditions may be milder in reality.

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