Crossing Network versus Dealer Market: Unique Equilibrium in the Allocation of Order Flow *

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Abstract

The allocation of order flow between different market venues can be characterized as a coordination game with multiple equilibria. We reconsider the allocation of order flow between a crossing network and a dealer market and show that either small differences in traders' information or a sufficient mass of traders with low liquidity preference generate a unique equilibrium, in which patient traders use the crossing network while impatient traders submit orders directly to the dealer market. Our model suggests that assets with low price volatility and large turnovers are likely to be traded on crossing networks, while others are traded on dealer markets.

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1 Introduction

Inter–market competition for order flow has been growing significantly in recent years. Key forces are the deregulation and globalization of financial markets. Technological progress in communication technologies and a reduction in communication costs enable customers to access different trading venues independent of their physical location. The automation of trading has strengthened the popularity of alternative trading systems, such as crossing networks (CNs) and electronic communication networks (ECNs). These networks attract order flow away from existing exchanges by offering lower commission prices and the longest after–hours trading. Thereby, they increase inter–market competition for order flow. One of the major concerns with respect to multi–market trading is the question, how liquidity is allocated between different market venues. In order to provide a liquid platform, new trading venues must attract sufficient order flow to ensure a high probability of order execution.

Typically, electronic markets are first established as CNs where market orders are carried out at a price adopted from traditional markets, in which dealers set prices to equalize supply and demand. Electronic markets are most successful in attracting order flow for homogeneous products with large turnover. Once established, many CNs expand by allowing limit orders and introduce their own price discovery. Then, they are classified as ECNs. In financial markets, we observe many hybrid market structures, in which CNs or ECNs are allied with traditional dealer markets. Here, price discovery results from orders submitted to both market venues and orders that cannot be executed in the electronic market are eventually routed to a dealer market for best–price execution.

In this paper, we focus on the allocation of order flow between traditional dealer markets and a CN. Trading on established dealer markets guarantees immediate order execution at bid and ask prices quoted by market makers. Trading on the electronic market is less expensive, as traders do not have to pay for an intermediary's services but only a small commission. However, the execution of an order submitted is uncertain. The probability of execution depends on the number of orders submitted. As more traders direct their orders to the electronic matching market, liquidity increases and raises the probability of execution for all submitted orders. This, in turn, attracts even more traders to submit their orders to the electronic market.

The allocation of order flow can be understood as a coordination game with positive network externalities. If and only if many traders coordinate to trade on the electronic market, the probability of order execution is high and expected payoff from trading on this market exceeds the payoff from trading on dealer markets. Coordination failure may result in the immediate failure of the new market due to a lack of liquidity. This raises the question under which circumstances and for which sort of assets or commodities electronic matching markets can co–exist with dealer markets or even replace them. Related to these are the questions which parameters influence the traders' decision where to trade and whether low trading cost are a sufficient condition for concentration of trade on an electronic crossing network.

We show that traders with a high preference for immediate order execution prefer trading at traditional dealer markets, while patient traders prefer submitting orders to a CN. If traders's beliefs about the value of immediate order execution are private information, there is a unique equilibrium, in which assets with large turnover and low average liquidity preference will be traded at the CN. Assuming that liquidity preference is positively related to price volatility, our model predicts that assets with large turnover and low price volatility are more likely being traded at electronic markets, while others are traded at dealer markets.

Previous theoretical research that addresses the allocation of order flow between market venues suffers from an indeterminacy due to multiple equilibria of the underlying coordination game. Pagano (1989) examines competition between two centralized markets and between a centralized market and direct search for a trading partner. He focuses on the role of traders' beliefs about the actions of other traders and their impact on markets' performance. There is no intermediary but liquidity arises as a function of scale. Depending on the transaction cost differential between markets, multiple rational expectation equilibria arise. If markets have identical transaction costs, the equilibrium in which both markets exist is unstable and trade will rather concentrate in one of them. If markets differ in transaction costs or in the search mechanism there may be either fragmentation or consolidation of trading, depending on the traders' initial expectations about other traders' decisions where to trade. When there is fragmentation, smaller traders go to the less expensive but illiquid market and larger traders to the more expensive but liquid market.

Considering competition between a centralized market with an intermediary that offers guaranteed execution and a decentralized search market where heterogeneous liquidity traders meet randomly and negotiate prices, Gehrig (1993) analyzes how the intermediaries' pricing behavior is affected by the existence of the search market and the employed bargaining process. He shows that there is an equilibrium in which traders with large gains from trade choose to trade with the monopolistic intermediary while traders with low gains from trade enter the search market. Here, both markets co–exist and order flow is fragmented.

Glosten (1994) examines an idealized electronic limit order book and shows that it does not invite competition from other markets while other markets do. Parlour and Seppi (2003) present a model of competition for order flow between different pairings of pure limit order markets and hybrid specialist/limit order markets. These models jointly describe liquidity demand and supply by assuming different types of traders: limit order traders and intermediaries supply liquidity while market order traders demand liquidity. Viswanathan and Wang (2002) analyze the traders' choice between a limit-order book, a dealership market and a hybrid market structure of the two when traders differ in size and risk aversion.

Chowdhry and Nanda (1991) analyze how the ability of traders to choose the trading venue affects functioning and liquidity of markets in the presence of informational asymmetries and liquidity traders, who are not allowed to switch to another market. They show that the market with the largest number of liquidity traders attracts liquidity and informed traders, resulting in a concentration of trading in this market. Hendershott and Mendelson (2000) study the impact of an introduction of a passive crossing network (CN) on traders and competitive dealer markets. When there are different types of heterogeneous liquidity and informed traders, liquidity creates a positive network externality, while informed traders enter the market exclusively on one market side and, thus, create a negative congestion effect. The most plausible equilibria share the property that low liquidity preference traders use the CN exclusively, traders with medium net gain use dealer markets when they could not be matched at the CN, and high liquidity preference traders go to the dealer market directly. Hendershott and Mendelson analyze comparative statics for some equilibria and argue that dealers' spreads are increasing in the proportion of traders, who use dealer markets opportunistically. Traders, who use the CN exclusively, generate the opposite effect.

These models suggest that markets with different trading costs and market structures may co–exist. They have one feature in common: there exist multiple equilibria resulting from the coordination problem among traders. Whether order flow is consolidated on one market or fragmented depends on the initial beliefs of traders about other traders' behavior. Given fragmentation of order flow, a sudden change in initial beliefs can result in traders switching markets and trade concentrating on one single market.

Our paper reconsiders the allocation of order flow between a CN and a dealer market and derives conditions for a unique equilibrium. Concentrating on liquidity traders, we show that either small differences in traders' information or a sufficient mass of traders with low liquidity preference generate a unique equilibrium.

In Section 2, we develop a model of liquidity–based competition between a pure dealer market and a CN. Traders' choices of market venues depend on transaction costs, probabilities of order execution and expected losses from unexecuted orders that may be interpreted as liquidity preference or as a measure of an asset's price volatility. If these losses are the same for all traders and common knowledge among them, multiple equilibria exist. In Section 3, we analyze the common knowledge game as a reference scenario. We compare efficiency and stability of equilibria and calculate critical market shares that are necessary for a CN to drive out dealer markets.

In Section 4, we allow expected disutilities of unexecuted orders to differ across individuals (private value game). In each equilibrium there is a threshold such that all traders with lower disutilities submit orders to the CN, while traders with higher disutilities trade on the dealer market. There is a unique equilibrium, if there is a sufficient mass of traders, for whom submitting orders to the CN is a dominant strategy. By attracting new traders who would not have submitted orders to a dealer market at the same point in time, a CN creates a minimal liquidity that is sufficient to attract even some traders, who would have gone to the dealer market otherwise. Thus, providing services that attract new customers and raise the overall market thickness is crucial to establish a CN in an environment where liquidity preferences differ across traders.

In Section 5 we adopt the global-game approach of Carlsson and van Damme (1993): we return to a common value for the gain from immediate order execution, but introduce noisy private information about this gain. If the noise in private information is sufficiently small, there exists a unique equilibrium with a threshold signal up to which agents submit orders to the CN. Agents who estimate the loss from unexecuted trade to be higher than this threshold trade on the dealer market. The threshold rises with rising bid–ask spread at the dealer market, with rising market thickness and with falling trading costs in the CN. If the expected disutility from an unexecuted order is proportional to an asset's price volatility, this model predicts that assets with high price volatility or small trading volume are exclusively traded on dealer markets, while assets with low price volatility or large turnover are traded in a CN.

In Section 6 we discuss the robustness of our results with respect to (i) a combination of private values and private information, (ii) endogenous trading costs, and (iii) price discovery in the electronic market.

Section 7 concludes the paper and gives an outlook on future research. An appendix contains all formal proofs.

2 Traders and Markets

Several features of market intermediation contribute to economies of scale: in a CN, a larger volume of trade raises the probability of finding trading partners with whom a trade can be carried out. In a traditional dealer market (DM), the market maker faces inventory risk that declines with the volume of trade and, thereby, allows lowering trading costs and attract even more traders. In addition, costs of price discovery and the risk of exploitation by insiders are decreasing in the proportion of orders that are directed to the same market venue. By these features, the choice of a market venue is a coordination game with strategic complementarities. The difference in expected payoffs from directing an order to some market A versus market B is a monotone function of the relative size of both markets.

Here, we use a stylized model in which strategic complementarities arise only from the probability of order execution at the CN, while trading costs at the DM and asset prices are treated as given exogenously. The other features would strengthen strategic complementarities without affecting our central results as will be explained in Section 7.

There is a continuum of agents $i \in [0, 1]$, from which a finite number of active traders is selected randomly for both sides of the market. The number of agents who are selected as buyers is denoted by N_b, the number of sellers by N_s. We assume that N_b and N_s are independently and identically distributed with $\text{prob}(N_b \neq N_s) > 0$. Furthermore, we assume that all agents have the same probability of being selected. Each trader can decide to buy [sell] one unit of the one and only asset either at a dealer market (DM) or at a crossing network (CN).

In the DM, traders trade with market makers who set bid and ask prices at which they are willing to buy or sell the asset. We normalize the mid–point of bid and ask price to zero, so that traders can buy the asset at price t_{DM} and sell at $-t_{DM}$, where t_{DM} is half of the bid–ask–spread and sometimes referred to as the DM's transaction fee.

The electronic CN offers purely transactional services without any intervention by an intermediary. Orders can be submitted to the CN as market orders and are executed at the mid–point between bid and ask price observed on the DM, i.e. zero. If an order is executed in the CN, the trader pays a small fee $t_{CN} < t_{DM}$.

There may be an imbalance of orders on the two sides of the CN, in which case the excess side is rationed stochastically. In this case, one runs the risk of an order not being executed. Orders on the excess side are randomly selected to match orders on the short side. The probabilities with which orders are executed are determined by the numbers of buyers and sellers who place their orders in the CN, denoted by n_b and n_s. The probability of a buy order in the CN to be executed is

$$\pi_b = \min\{1, n_s/n_b\}. \tag{1}$$

The probability of a sell order being executed is

$$\pi_s = \min\{1, n_b/n_s\}. \tag{2}$$

Unexecuted orders may be submitted to the market in the next period or passed to a dealer automatically. In either case, trades are executed with delay and possibly at a different price. Traders' choices of market venues depend on transaction costs, probabilities of order execution and expected losses from unexecuted orders. Such losses may arise from traders' impatience or urgency to trade or from the risk associated with an asset's price volatility. Higher price volatility is associated with higher risk of losing gains from trade and, thereby, increases expected losses from unexecuted orders. These losses may also depend on the costs of passing orders to another trading venue and on the length of the time interval until the order can be resubmitted. We consider a reduced form one–shot game, in which unexecuted orders leave the trader with some disutility $\theta \in [\check{\theta}, \hat{\theta}]$. Parameter θ is also referred to as liquidity preference.

The difference in expected payoffs from submitting an order to the CN instead of the DM is

$$\mathrm{E}\left(\pi\left(\theta - t_{CN}\right) - \left(\theta - t_{DM}\right)\right). \tag{3}$$

The model and its parameters are assumed to be common knowledge. For the disutility of unexecuted orders θ we consider three cases: first, we assume that all traders face the same disutility θ if an order remains unexecuted. In the next section, we assume in addition that θ is common knowledge. This assumption leads to multiple equilibria for a wide range of values.

In Section 4 we analyze equilibria of a game with private values θ^i. Disparate liquidity preferences may result from, for example, idiosyncratic endowments or (time) preferences. In general, θ^i may also be influenced by a trader's risk aversion, idiosyncratic beliefs or inside information. If the distribution of private values is common knowledge, a unique equilibrium requires that liquidity preferences are spread over a wide range with a sufficiently large share of agents for whom the CN is a dominant strategy. The unique equilibrium is associated with a critical value θ^*, such that all agents with smaller values submit their orders to the CN, while agents with higher values go to the DM. However, if the distribution of liquidity preferences is more concentrated, multiple equilibria persist.

In Section 5 we go back to the extreme case where all traders face the same disutility. Instead, we apply the global–game approach and introduce small noise in the observation of θ, so that agents only have private information about this variable. One possible interpretation may be that traders lack perfect information on the assets's price volatility or, at least, doubt that other agents interpret data in the same way. As a result, traders' expected losses from unexecuted orders are clustered around θ without being identical. As opposed to clustered private values with common knowledge of their distribution, here the posterior distribution of beliefs is private information and agents are uncertain about their relative position in this distribution.

Optimization under uncertainty establishes an additional equilibrium condition that leads to a unique equilibrium if the variance of private signals is sufficiently small. In this case, there exists a unique threshold signal x^* such that traders with smaller signals use the CN while traders with higher signals go to the DM. Similar results can be obtained with ideosyncratic liquidity preferences, when their distribution is private information. This will be discussed in Section 6.

To our knowledge, this is the first paper that compares the global–game approach, in which uniqueness of equilibrium arises from small differences in agents' information, with a private value game, in which uniqueness requires a sufficient dispersion of agents' payoffs.

3 Common Knowledge Game

When θ is the same for all agents and common knowledge, an individual strategy is a function a^i assigning either market to each θ conditional on whether the agent is selected as buyer or seller. $a^i(\theta, b) = 1$ means that agent i goes to the CN if she is a buyer and the value of trade is θ. If she is a seller, she goes to the CN if $a^i(\theta, s) = 1$. The total numbers of buyers and sellers, N_b and N_s, are unknown to traders, so that they always face some uncertainty about successful execution of an order placed at the CN.

Given a strategy combination $a = (a^i)_{i\in[0,1]}$ and disutility θ, the proportions of agents who submit orders to the CN if selected as buyers or sellers, respectively, are

$$\alpha_b(\theta, a) = \int_0^1 a^i(\theta, b)\,\mathrm{d}i \qquad \text{and} \qquad \alpha_s(\theta, a) = \int_0^1 a^i(\theta, s)\,\mathrm{d}i \tag{4}$$

Given these proportions and the random process that selects the number of buyers and sellers, the probabilities of order execution are well defined. We demonstrate this using a particular distribution assumption.

Assume that N_b and N_s have a geometric distribution with $\mathrm{E}(N_.) = \lambda$. The geometric distribution follows from the idea that there is an infinite set of agents out of which potential buyers and sellers are selected randomly. With probability γ a first agent is selected as buyer. A second buyer is selected with probability γ if and only if another buyer has been selected already. Thus, the probability of having at least n buyers is γ^n and the total number of buyers N_b has a geometric distribution with an expected number of $\lambda = \frac{\gamma}{1-\gamma}$. The same procedure is applied to select sellers.

Lemma 1 *Suppose a fraction α_b of all traders goes to the CN if selected as buyers and a fraction α_s of all traders goes to the CN if selected as sellers. If N_b and N_s are independently drawn from a geometric distribution with $E(N_.) = \lambda$, the probability with which a buy order at the CN is executed is given by*

$$\Pi(\alpha_b, \alpha_s) = \frac{\alpha_s}{\alpha_b} \ln\left(1 + \frac{\alpha_b\,\lambda}{1 + \alpha_s\,\lambda}\right).$$

The probability of execution of a sell–order is $\Pi(\alpha_s, \alpha_b)$, accordingly.

Lemma 1 is a generalization of a result by Hendershott and Mendelson (2000, Proposition 3, p. 2081) who prove the special case of $\alpha_b = \alpha_s$. All proofs are given in the

Appendix. For a buyer the expected payoff from going to the CN instead of going to the DM is

$$\tilde{U}_b(\theta, a) = (\theta - t_{CN})\,\Pi(\alpha_b(\theta, a), \alpha_s(\theta, a)) - (\theta - t_{DM}) \tag{5}$$

and for a seller accordingly

$$\tilde{U}_s(\theta, a) = (\theta - t_{CN})\,\Pi(\alpha_s(\theta, a), \alpha_b(\theta, a)) - (\theta - t_{DM}). \tag{6}$$

Let a^* be a Nash equilibrium. In equilibrium traders go to the market with higher expected payoff. Thus,

$$\alpha_s(\theta, a^*) = \begin{cases} 1 & \text{if} \quad \tilde{U}_b(\theta, a^*) > 0 \\ 0 & \text{if} \quad \tilde{U}_b(\theta, a^*) < 0 \end{cases} \tag{7}$$

and

$$\alpha_b(\theta, a^*) = \begin{cases} 1 & \text{if} \quad \tilde{U}_s(\theta, a^*) > 0 \\ 0 & \text{if} \quad \tilde{U}_s(\theta, a^*) < 0 \end{cases} \tag{8}$$

To analyze equilibria, we first show that the same proportions of buyers and sellers submit orders to the CN.

Lemma 2 *If a^* is a Nash equilibrium of the common knowledge game, then*

$$\alpha_b(\theta, a^*) = \alpha_s(\theta, a^*) = \alpha(\theta, a^*) \qquad \textit{for all } \theta.$$

If different proportions of buyers and sellers submit orders to the CN, their payoffs differ as well, which contradicts the equilibrium property that traders choose the trading venue that promises the highest expected payoff. Associated with each Nash equilibrium a^* is a market share for the CN of $\alpha(\theta, a^*)$. Given the geometric distribution for the number of traders, the probability of order execution is

$$\tilde{\pi}(\alpha) = \ln\left(1 + \frac{\alpha\,\lambda}{1 + \alpha\,\lambda}\right). \tag{9}$$

$\tilde{\pi}$ is strictly increasing in α up to

$$\bar{\pi} = \tilde{\pi}(1) = \ln\left(1 + \frac{\lambda}{1 + \lambda}\right) < 0.7. \tag{10}$$

For $\lambda \to \infty$, the probability of order execution converges to $\ln(2) \approx 0.693$.

It is a general property of finite markets that the probability of order execution is bounded away from 1 even if all potential traders decide for the CN. The numbers of buyers and sellers are drawn independently and, therefore, it is a rare event that the numbers on both sides of the market are equal, even when the expected number of traders on each side is extremely large. Accordingly, each trader faces some positive probability of being rationed whenever the expected number of traders is finite. This property has often been neglected by assuming non–atomistic traders as in Gehrig (1993).

If disutilities from unexecuted orders are extremely large, then the risk at the CN can never compensate the low fees. Even if a trader believes that all other traders submit their orders to the CN, for a sufficiently high θ, the DM is a dominant strategy. The frontier of this dominance region is given by θ_0, for which both market venues promise the same expected payoff, provided that all traders go to the CN. θ_0 is defined by

$$(\theta_0 - t_{CN})\,\bar{\pi} = \theta_0 - t_{DM} \qquad \Leftrightarrow \qquad \theta_0 = \frac{t_{DM} - \bar{\pi}\, t_{CN}}{1 - \bar{\pi}} \tag{11}$$

Note that for the geometric distribution with $\lambda \to \infty$, this value converges to $\frac{t_{DM} - \ln(2)\, t_{CN}}{1 - \ln(2)}$ and not to infinity. For other distributions on the number of traders, θ_0 is also converging to some finite number if the expected number of traders rises to infinity.

Equilibrium strategies are individually optimal, given that all other agents play the strategies of the same equilibrium. For $\theta > \theta_0$ the payoff on the DM exceeds the expected payoff in the CN, even if all others traders coordinate to use the CN. Given the high demand for immediacy, even the lowest possible risk of an order not being executed in the CN outweighs the difference in fees. Here, trading on the DM is a dominant strategy and the only equilibrium.

For $t_{CN} < \theta < t_{DM}$, agents lose from trading on the DM and expect profits in the CN. Here, trading in the CN is a dominant strategy and the only equilibrium.

For trading values $\theta \in [t_{DM}, \theta_0]$ there are multiple equilibria. In one equilibrium, all agents go to the DM. A single trader cannot gain by switching to the CN, because without trading partner her order would not be executed. In another equilibrium, all agents go to the CN. Their expected payoff is not lower than at the DM and, hence, no agent has an incentive to leave the CN.

In addition to these pure equilibria, there are mixed equilibria, in which both markets co–exist with a market share for the CN of $\tilde{\alpha}(\theta)$ that increases from zero to one as θ rises from t_{DM} to θ_0. In these equilibria, the market share of the CN just generates an execution probability for which expected payoffs at both markets are equal, so that no agent wants to switch. If expected payoffs are the same on both markets, traders in the CN cannot gain from switching to the DM. On the other hand, a single trader, say buyer, who switches from DM to CN, would reduce $\mathrm{E}(\pi_b)$ and, thus, would expect smaller gains from trade in the CN. Mixed equilibrium market share $\tilde{\alpha}(\theta)$ is given by

$$\tilde{\pi}(\tilde{\alpha})\,(\theta - t_{CN}) = (\theta - t_{DM}) \quad \Leftrightarrow \quad \tilde{\alpha}(\theta) = \tilde{\pi}^{-1}\left(\frac{\theta - t_{DM}}{\theta - t_{CN}}\right). \tag{12}$$

Nash equilibria are summarized in Theorem 1 and illustrated in Figure 1.

Theorem 1 *A strategy combination a^* is a Nash equilibrium of the game with common knowledge if and only if*

$$\begin{aligned}
&\alpha_b(\theta, a^*) = \alpha_s(\theta, a^*) = 1 && \textit{for} \quad \theta \in (t_{CN}, t_{DM}),\\
&\alpha_b(\theta, a^*) = \alpha_s(\theta, a^*) \in \{0, \tilde{\alpha}(\theta), 1\} && \textit{for} \quad \theta \in [t_{DM}, \theta_0],\\
&\alpha_b(\theta, a^*) = \alpha_s(\theta, a^*) = 0 && \textit{for} \quad \theta > \theta_0.
\end{aligned}$$

For a geometric distribution of market size

$$\tilde{\alpha}(\theta) = \frac{1}{\lambda} \cdot \left[\exp\left(\frac{\theta - t_{DM}}{\theta - t_{CN}}\right) - 1\right] \cdot \left[2 - \exp\left(\frac{\theta - t_{DM}}{\theta - t_{CN}}\right)\right]^{-1}.$$

Insert Figure 1 here

Figure 1 *Nash equilibria of the common knowledge game. For $t_{CN} = 1$, $t_{DM} = 2$ and $\lambda = 15$, the upper limit of the multiplicity region is $\theta_0 = 3.953$, while $\bar{\pi} = 0.6614$.*

Which of the three types of equilibria should one expect in real situations of inter–market competition? One possible answer to this question is provided by the relative stability of equilibria. In a Nash equilibrium no *single* player has an incentive to deviate. But, intermediaries may compete by attracting a large number of traders at the same time, e.g. by offering a temporary discount on transaction fees or by rewarding traders who submit orders to their venue.

An equilibrium is called ‘strong equilibrium’, if no *coalition* of players can improve upon their payoffs by a coordinated deviation from this equilibrium. If all traders go to the CN, they receive a higher payoff than at the DM, provided that $\theta < \theta_0$. Any coordinated deviation by many traders would leave the remaining customers of the CN with a lower payoff, but deviators would not gain at the DM. Market consolidation in the CN is a strong equilibrium. Therefore, it is a hard task for a dealer to attract order flow away from a CN that has already captured the market. To intrude a CN–monopoly, the dealer has to compensate all traders by lower fees until the CN is so drained out that lower transaction costs do not compensate for execution risk and the DM is more attractive even with higher fees.

If all traders go to the DM, it needs a coalition of at least size $\tilde{\alpha}(\theta)$ switching to the CN, to improve upon their payoffs. If θ is just above t_{DM}, traders make a small gain at the DM. A small group of traders joining the CN creates a liquidity sufficient to raise expected payoffs at the CN above the payoff at the DM. The DM–equilibrium is rather weak here, and it is easy for a CN to attract the initial order flow that is necessary to become more attractive than the DM. With rising θ the DM–equilibrium becomes stronger as coalitions of growing size are needed to raise the execution probability in the CN to a level that outweighs the difference in fees. A CN must manage to coordinate a proportion of at least $\tilde{\alpha}(\theta)$ of traders in order to intrude a DM–monopoly. This is very difficult in markets with a high θ, because $\tilde{\alpha}(\theta)$ rises exponentially. For $\theta \geq \theta_0$, the DM–equilibrium is strong and cannot be improved upon by any coalition.

The mixed equilibrium is very weak. Expected payoffs are the same in both markets and any coalition with at least one trader on each side of the market can improve their payoffs by switching from DM to CN. A coalition switching from CN to DM cannot improve. Therefore, once a CN has a market share of $\tilde{\alpha}(\theta)$, one should expect that it takes over the whole market. Critical mass $\tilde{\alpha}$ rises in θ and t_{CN} and falls in t_{DM}, as these changes reduce the relative advantage of the CN. A rise in the expected number of traders λ, i.e. rising “thickness” of the market, increases probability of order execution and makes the CN more attractive. This lowers $\tilde{\alpha}(\theta)$ and raises θ_0.

With respect to the situation where existing DMs face upcoming electronic CNs, we suggest the following interpretation: the lower the expected value of immediate trade θ, the easier is it to intrude the market with a CN. Intrusion is possible only if $\theta \leq \theta_0$. For $\theta > \theta_0$ agents trade on the DM exclusively. While it is increasingly easy for an intruding CN to win the market when θ goes down, it becomes more and more costly for market makers to re–attract order flow. To convince traders to return to the DM, they must be compensated for higher bid–ask spreads. This, in turn, is costly for market makers. It pays for the DM to protect itself against attempts to establish a CN–monopoly. A CN may fail to enter the market if its market share stays below the critical value $\tilde{\alpha}(\theta)$.

For trading values $\theta \in [t_{DM}, \theta_0]$ multiple equilibria exist. Although these equilibria differ in strength, it is not possible to predict at which market trade consolidates. The CN–equilibrium is the only strong equilibrium, but, it is hard to imagine that a CN may achieve full market share in situations when coordination of almost all traders is required to provide sufficient liquidity. In laboratory experiments, Van Huyck, Battalio and Beil (1990) and Chaudhuri, Schotter and Sopher (2001) demonstrate that it is almost impossible to achieve efficient coordination in such games if the group size exceeds ten subjects. Traders may be locked in an inefficient equilibrium due to a lack of coordination. Multiple equilibria open ways for self–fulfilling beliefs and inertia effects. Once a market has been established as main trading venue, braking the monopoly may require tremendous funds to attract the necessary liquidity and keep it until the original monopolist is too exhausted to fight back.

Mixed equilibria are extremely unstable and strategy combinations that are no Nash equilibrium will not survive in the long run either. We would therefore expect that all traders direct their orders to the same market venue. On the other hand, orders that are not executed in the CN may be passed on to a dealer in the next period. Then, the dealer gets at least the leftovers, even when initially all traders direct orders to the CN. The expected proportion of orders that are not executed in the CN is $1 - \bar{\pi}$ which is greater than 0.3 in case of the geometric distribution. Thereby, an automatic routing of unexecuted orders from CN to DM provides dealers with a minimum liquidity justifying quotations of bid and ask prices on which the CN is a free rider. This implies, however, that dealers are taking inventory, because all orders are from the same side of the market and even out only over time.

4 Private Value Game

Multiplicity of equilibria hinges on the assumption of a common value θ that is common knowledge to all agents. In this section we show that the equilibrium may be unique, if liquidity preferences differ across individuals.

Each agent i has a private value of immediacy θ^i. The distribution of private values is defined by a density function $f(\theta^i)$. To simplify exposition, we identify agents with their preferences, i.e. $i = \theta^i$. We assume that $\theta^i \geq t_{CN}$ for all $\theta^i \in \text{supp}(f)$.

Otherwise, we would need to consider “no trade” as a third option.

Distribution f is common knowledge, and each trader knows her own preference θ^i. However, she does not know how many other traders are selected as buyers and sellers and what their preferences are. Traders for both market sides are selected according to the random process described in the last section. After selection, a trader decides to which market she submits her order. A strategy is a function $a^i : \{b, s\} \to \{0, 1\}$, where $a^i(b) = 1$ means that trader i goes to the CN if she is selected as buyer.

Let $\alpha_b(a)$ [$\alpha_s(a)$] be the expected proportion of traders who use the CN if they are selected as buyers [sellers]. For any given strategy combination a, the probability of execution is $\pi_b(a) = \Pi(\alpha_b(a), \alpha_s(a))$ for a buy order and $\pi_s(a) = \Pi(\alpha_s(a), \alpha_b(a))$ for a sell order.

Given strategy combination a, the expected payoff for trader i going to the CN instead of the DM is

$$\tilde{U}_b(\theta^i, a) = (\theta^i - t_{CN})\, \pi_b(a) - (\theta^i - t_{DM}) \tag{13}$$

if i is a buyer and

$$\tilde{U}_s(\theta^i, a) = (\theta^i - t_{CN})\, \pi_s(a) - (\theta^i - t_{DM}) \tag{14}$$

if i is a seller. Since the probability of order execution is bounded below 1, $\tilde{U}_b$ and $\tilde{U}_s$ are strictly decreasing in θ^i for any strategy combination a. Using this property, we can show that in each equilibrium there is a value θ^*, such that all agents with private values below θ^* submit their orders to the CN, while agents with private values above θ^* trade on the DM.

Lemma 3 *If a^* is a Nash equilibrium of the private value game, there is a unique $\theta^*(a^*) \in [t_{DM}, \theta_0]$, such that*

$$a^{*i}(b) = a^{*i}(s) = \begin{cases} 1 & \text{if} \quad \theta^i < \theta^*(a^*) \\ 0 & \text{if} \quad \theta^i > \theta^*(a^*) \end{cases}$$

The associated proportion of agents who submit orders to the CN is given by $\alpha^* = F(\theta^*)$, where F is the cumulative distribution of private values. The probability of order execution is $\tilde{\pi}(\alpha^*)$, and excess utility from going to the CN instead of the DM is

$$\hat{U}(\theta^*) = (\theta^* - t_{CN})\, \tilde{\pi}(F(\theta^*)) - (\theta^* - t_{DM}). \tag{15}$$

In equilibrium $\hat{U}(\theta^*) = 0$, and any zero point of $\hat{U}$ describes an equilibrium in which agents use the CN if an only if their liquidity preferences are below θ^*. If there is a sufficient proportion of traders, for whom the CN is a dominant strategy, there is a unique equilibrium, as described by the following theorem.

Theorem 2 *Suppose that traders are selected by a geometric distribution and private values have a uniform distribution in* $[t_{CN}, \hat{\theta}]$. *If* $\frac{t_{DM}-t_{CN}}{\hat{\theta}-t_{CN}}\lambda \geq \frac{1}{2}$, *the private value game has a unique Nash equilibrium. If, in addition,* $\hat{\theta} > \theta_0$ *the equilibrium is associated with an interior threshold* $t_{DM} < \theta^* < \theta_0$. *If, instead,* $\hat{\theta} \leq \theta_0$, *then* $\theta^* = \theta_0$ *and (almost) all traders submit orders to the CN.*

For a unique equilibrium with an interior threshold, there must be some agents, with private values above θ_0 and a sufficient mass of agents with private values below t_{DM}. Theorem 2 requires that the expected number of these agents is at least 1/2. This guarantees a minimal probability of order execution $\tilde{\pi}(\frac{t_{DM}-t_{CN}}{\hat{\theta}-t_{CN}}) \geq \ln(4/3) \approx 0.288$. Given this probability at the lower end of $[t_{DM}, \theta_0]$, the increase in π that is associated with a rising threshold value θ is too weak to compensate for the increasing disadvantage of the CN stemming from uncertainty of gain θ. Thus, $\hat{U}(\theta^*)$ is monotonically decreasing at θ^* and equilibrium is unique.

If disutilities of unexecuted orders differ sufficiently between traders to have some traders for whom going to the DM is a dominant strategy and a sufficient mass of traders for whom going to the CN is a dominant strategy, then there is a unique equilibrium with a threshold θ^*, such that all traders with lower disutilities submit orders to the CN while traders with higher values go to the DM. Under these conditions, initial orders are split between both markets. The market share of the CN rises with rising t_{DM} and falling t_{CN}. Figure 2 illustrates a unique Nash equilibrium of this kind.

Insert Figure 2 here

Figure 2 *Nash equilibrium of the private value game. An intersection of the two curves at* θ^* *represents a Nash equilibrium if all agents with private values below* θ^* *use the CN and agents with private values above* θ^* *go to the DM. For* $t_{CN} = 1$, $t_{DM} = 2$, $\lambda = 15$ *and* $\hat{\theta} = 10$, *we get* $\theta_0 = 3.953$ *and* $\theta^* = 3.433$. *The share of initial order flow submitted to the CN is* $F(\theta^*) = 27\%$. *Execution probability is* $\tilde{\pi}(F(\theta^*)) = 59\%$.

More important than the share of orders initially submitted to a market is the share of orders that are executed there. If unexecuted orders are passed on to the DM, the share of trades executed in the CN is $\tilde{\pi}(F(\theta^*)) \cdot F(\theta^*)$, which is only 16% in the example of Figure 2.

If the support of f is a subset of (t_{DM}, θ_0), there are at least three Nash equilibria comparable to those in the common knowledge game: either initial orders concentrate on one of the two markets or initial order flow is fragmented, with some interior θ^*. In general, multiplicity of equilibria requires that there are very few traders with private values below t_{CN}. An example with a truncated normal distribution is shown in Figure 3.

Insert Figure 3 here

Figure 3 *Multiple Nash equilibria in the private value game with a truncated normal distribution of private values, where* $E(\theta) = 3.5$ *and* $Var(\theta) = 0.25$. *Truncations are at* $\check{\theta} = t_{CN} = 1$ *and* $\hat{\theta} = 10$. $t_{DM} = 2$. *There are three equilibria at* $\theta_1 = 2.023$, $\theta_2 = 2.828$ *and* $\theta_3 = 3.880$. *Associated shares of initial order flow submitted to the CN are 0.2%, 8.9% and 77.6%.*

If there are three equilibria, the equilibrium with the largest share of the CN is strong. The equilibrium with the highest market share of the DM is robust against deviations of any small coalition. The "middle" equilibrium is unstable: if some agents switch from the DM to the CN, they attract others with values just above θ^*, who attract more agents with even higher values, and so on. With best–reply learning agents should converge to the equilibrium with the highest market share of the CN. Vice versa, if some agents originally switch from the CN to the DM. However, best reply learning needs many periods, while in the common value game, the best reply of all agents is identical, and once the market share of the CN deviates from $\tilde{\alpha}$ all agents jump to the winning market immediately.

If there is a unique equilibrium, for example under conditions of Theorem 2, individual decisions neglect positive external effects on the execution probability in the CN. Efficiency considerations depend on whether private values are information based and randomly assigned to agents after their selection as traders or whether these values are inherent properties of agents' preferences, e.g. stemming from risk aversion. In the latter case, and if utility is not transferable across agents, any allocation different from equilibrium reduces expected payoffs for some agents. If execution risk can be hedged by transferable payoffs, the efficient threshold θ^{**} maximizes the sum of individual payoffs weighted with respective probabilities of the agents' participation given by the density function f. The same holds if private values are randomly assigned to agents. Here, f is the density function of the information based private value for each agent. The efficient threshold θ^{**} maximizes the expected payoff with respect to distribution f.

Theorem 3 *If the cumulative density F is continuous and the private value game has a unique equilibrium, associated with an interior threshold $\theta^* < \hat{\theta}$, then θ^* is smaller than θ^{**}.*

If prob$(\theta^i \in (\theta^*, \theta^{**})) > 0$, the market share of the CN is inefficiently small. Traders with private values between θ^* and θ^{**} submit their orders to the DM. If they would go to the CN instead, the overall gains for all traders induced by higher liquidity of the CN exceed the losses that traders in the CN must expect whose values are closest to θ^{**}.

Results of this game are related to Gehrig (1993), who also shows that traders with a low liquidity preference choose direct trading instead of trading with an intermediary. Gehrig's (1993) model differs from ours in that he does not consider a CN, but traders must search for partners by themselves and matching occurs with a rather low probability. Thus, execution probability is bounded above far from unity. In Gehrig's model, a whole continuum of traders is active on both sides of the market. Hence, there is no uncertainty about market size. Introducing a CN in Gehrig's model would guarantee multiple equilibria, and execution probability in the CN would either be zero or one. Uncertainty in Gehrig (1993) stems from the search process, while in our model uncertainty is due to the finite random selection of active traders. In both models, the probability of order execution at the direct market is limited — a crucial feature for uniqueness of the critical value θ^* that divides customers of the two markets. In equilibrium, uncertainty of order execution is a result of the matching technology at the direct market. Automation of trading and price discovery may raise the probability of order execution tremendously and increase the market share of direct markets accordingly.

The results in this section are also related to those of Herrendorf, Valentinyi and Waldmann (2000) who study multiplicity and indeterminacy in two–sector models with sector–specific labor and positive externalities. In their model, individuals differ in productivity and choose the sector in which they work. Herrendorf, Valentinyi and Waldmann show that enough heterogeneity in agents' sector–specific productivity can ensure uniqueness of the chosen stationary state as it prevents sufficiently many agents from changing their choice in reaction to a change in beliefs about the production of the sectors. This is in line with our result that a sufficient mass of traders with private values below t_{DM} ensures a unique equilibrium.

5 Private Information Game

As we have seen in the last section, different liquidity preferences across individuals are, in general, not sufficient for equilibrium uniqueness, e.g. if the distribution of preferences is concentrated in the intermediate region. In this section, we go back to the case of a common value θ, introduced in Section 3. As we have seen above, this is an extremely unfavorable case for achieving uniqueness. However, multiplicity of equilibria in Section 3 is not only due to a common value, but also to common knowledge of this value.

If this value represents the volatility of an asset's price, there is no reason to believe that all traders agree on their estimates of θ. Suppose that a trader is uncertain about θ and only has a private estimate (signal) x^i. When there is uncertainty about θ, there is also uncertainty about the estimates of other agents. Even if strategies are known, the actual behavior of other agents is uncertain to each trader. This uncertainty creates an additional restriction for equilibria that can be used to eliminate strategies that are equilibria under common knowledge of θ. Applying the global–game approach, introduced by Carlsson and van Damme (1993), extended by Morris and Shin (2003), and tested by Heinemann, Nagel and Ockenfels (2004a), we show that private information on θ may also lead to a unique equilibrium.

Assume that θ is a random variable with some distribution in $[\check{\theta}, \hat{\theta}]$. We assume once more that $\check{\theta} \geq t_{CN}$, so we do not need to consider the option of "no trade". Agents do not know the realization of θ, but only get some private signal $x^i = \theta + u^i$. Error terms u^i are independently and identically distributed around 0. For means of exposition, we assume that θ has a uniform distribution in $[\check{\theta}, \hat{\theta}]$ and u^i has a uniform distribution in $[-\epsilon, +\epsilon]$.

An individual strategy assigns either market to each possible signal. $a^i(x^i) = 1$ [0] means that agent i goes to the CN [DM] if her signal is x^i. Here, we think of traders choosing the market irrespective of their wish to buy or sell the asset. They decide on the market depending on their signal before they are selected as buyers or sellers. This must be taken into account for any interpretation of equilibria. The market that we consider has the same people trading on both sides. This may be a suitable assumption for many asset markets, but not for all. It is most certainly not appropriate for product markets, where buyers and sellers are firms of different branches, for retail markets or markets with participants who exercise market power. With this restriction, agents' decisions are strategic complements, which is a sufficient condition for the global–game approach yielding a unique equilibrium.[1]

[1]We believe that it is possible to extend the proof of our main theorem in this section to a

Denote the conditional density of signal x^i for given disutility θ by $f(x^i\,|\,\theta)$. The proportion of players who go to the CN if selected as buyers or sellers is

$$\alpha(\theta, a) = \int_{-\infty}^{\infty} \int_{0}^{1} f(x^i\,|\,\theta)\, a^i(x^i)\, \mathrm{d}i\, \mathrm{d}x^i. \tag{16}$$

The probability of order execution in the CN conditional on the value of trade θ and strategy combination a is $\tilde{\pi}(\alpha(\theta, a))$, and the expected payoff for agent i going to the CN instead of the DM is

$$\tilde{U}(x^i, a) = \mathrm{E}\Big((\theta - t_{CN})\,\tilde{\pi}(\alpha(\theta, a)) - \theta + t_{DM}\;\Big|\; x^i\Big). \tag{17}$$

The more traders direct their orders to the CN at some signals, the higher is the probability of order execution at all surrounding signals. $\tilde{U}(x^i, a)$ is non–decreasing in the mass of states and traders associated with orders going to the CN. Hence, market choices are strategic complements.

If the support of θ includes t_{DM} and θ_0 and the variance of u^i is sufficiently small, then there exist two signals $\underline{x}^0$ and $\bar{x}^0$, such that

$$E(\theta\,|\,\underline{x}^0) = t_{DM} \qquad \text{and} \qquad E(\theta\,|\,\bar{x}^0) = \theta_0. \tag{18}$$

For the uniform distribution, we assume that $\check{\theta} < t_{DM} - \epsilon$ and $\hat{\theta} > \theta_0 + \epsilon$. Then, $\mathrm{E}(\theta\,|\,x^i = t_{DM}) = t_{DM}$ and $\mathrm{E}(\theta\,|\,x^i = \theta_0) = \theta_0$, so that $\underline{x}^0 = t_{DM}$ and $\bar{x}^0 = \theta_0$.

Using (17), we find that for $x^i < \underline{x}^0$ expected payoff $\tilde{U}(x^i, a) > 0 \quad \forall a$. Thus, for an agent who gets a signal below t_{DM}, it is a dominant strategy to go to the CN. The trader expects a positive reward in the CN with some probability that depends on the strategies of other agents, while the expected reward on the DM is certainly negative.

For $x^i > \bar{x}^0$, we find $\tilde{U}(x^i, a) < 0 \quad \forall a$. Thus, for an agent who gets a signal above θ_0, it is a dominant strategy to go to the DM. The expected value of trade is so big that certainty of execution in the DM outweighs the lower costs in the CN even for the highest possible execution probability.

situation, where strategies may depend on players being selected as buyers or sellers. In equilibrium, strategies will be symmetric for almost all signals. But, traders on the same side of the market compete with each other for being matched in the CN, and their choices are no strategic complements. It is yet an open question, how the global–game approach can be generalized to games in which strategies are not complementary.

Having strategic complementarity and two dominance regions, we can find sequences $\{\underline{x}_k, \bar{x}_k\}_{k\in\mathbb{N}_0}$ by iteratively eliminating dominated strategies. As is well known, this procedure amounts to the assumption that players are rational and rationality is common knowledge. Starting with $k = 0$, rational players do not choose the DM for $x^i < \underline{x}^0$, nor the CN for $x^i > \bar{x}^0$. Hence we may eliminate these dominated strategies. When rationality is common knowledge an agent will not play a strategy that is dominated if she considers only those strategies of other players that have not been eliminated yet. At step $k+1$ agents consider only the strategies that assign the CN to signals below $\underline{x}^k$ and the DM to signals above $\bar{x}^k$.

The more agents decide for the CN the higher is the incentive for each agent to go to the same market. After eliminating dominated strategies, the best [worst] thing that can happen to a potential trader in the CN is that all other potential traders who have signals in $[\underline{x}^k, \bar{x}^k]$ go to the CN [DM]. This maximizes [minimizes] the probability of execution of an order in the CN. So, the best [worst] strategy combination that an agent at step $k+1$ must consider is given when all other agents play strategy $I_{\bar{x}^k}$ $[I_{\underline{x}^k}]$. Define a strategy I_y by

$$I_y(x^i) = \begin{cases} 1 & \text{if } x^i \leq y \\ 0 & \text{if } x^i > y. \end{cases} \tag{19}$$

In other words, an agent playing this strategy goes to the CN if and only if her signal is not bigger than y.

It is a dominant strategy to go to the CN whenever the lowest expected return there exceeds the certain return at A, i.e. $\tilde{U}(x^i, I_{\underline{x}^k}) > 0$. This is the case when $x^i < \underline{x}^{k+1}$, defined by

$$\underline{x}^{k+1} = \min\{x \,|\, \tilde{U}(x, I_{\underline{x}^k}) = 0\}. \tag{20}$$

It is a dominant strategy to go to the DM, if the highest expected return in the CN is lower than the gain at the DM. This happens for $x^i > \bar{x}^{k+1}$, defined by

$$\bar{x}^{k+1} = \max\{x \,|\, \tilde{U}(x, I_{\bar{x}^k}) = 0\}. \tag{21}$$

$\tilde{U}(x, I_y)$ rises in y. Since $\underline{x}^k < \bar{x}^k$, we have $\underline{x}^{k+1} < \bar{x}^{k+1}$. If rationality is common knowledge, players use the CN if they get signals below $\underline{x}^k$, and they go to the DM if their signals exceed $\bar{x}^k$ for any k. Common knowledge of rationality takes this procedure to the limits, where a trader with signal x^i will always use the CN if

$x^i < \underline{x}^\infty = \lim_{k\to\infty} \underline{x}^k$ and always go to the DM if $x^i > \bar{x}^\infty = \lim_{k\to\infty} \bar{x}^k$. Sequences $\underline{x}^k$ and $\bar{x}^k$ are monotone and bounded, so that limit points exist and are given by

$$\begin{aligned} \underline{x}^\infty &= \min\{x \,|\, \tilde{U}(x, I_x) = 0\} \\ \bar{x}^\infty &= \max\{x \,|\, \tilde{U}(x, I_x) = 0\}, \end{aligned} \tag{22}$$

where

$$\tilde{U}(x, I_x) = \mathrm{E}((\theta - t_{CN})\, \tilde{\pi}(F(x|\theta)) - \theta + t_{DM} \,|\, x), \tag{23}$$

and $F(x|\theta)$ is the cumulative density of signal x given value of trade θ. $\underline{x}^\infty$ and $\bar{x}^\infty$ are the limits of the set of strategies surviving the iterative elimination procedure.

From Milgrom and Roberts (1990) we know that in a game with strategic complementarities, the range of iteratively undominated strategies is limited by Nash equilibria. Indeed, it is easy to see that $I_{\underline{x}^\infty}$ and $I_{\bar{x}^\infty}$ are Nash equilibria of the private information game. Since Nash equilibria can never be eliminated, there is no Nash equilibrium where agents go to the DM for signals below $\underline{x}^\infty$ or to the CN for signals above $\bar{x}^\infty$.

Limit points $\underline{x}^\infty$ and $\bar{x}^\infty$ are the smallest and the largest solution of equation $\tilde{U}(x, I_x) = 0$. In general, this equation may have several solutions, so that we may be left with multiple equilibria, although we could clearly reduce the set of disutilities with unpredictable outcomes in comparison to the game with common knowledge of θ. However, if $\mathrm{d}\tilde{U}(x, I_x)/\mathrm{d}x$ is negative for each solution of $\tilde{U}(x, I_x) = 0$, then the equilibrium is unique. Signals enter this function in two ways. The partial derivative of $\tilde{U}$ with respect to x is negative. An increase in x increases expected return in the CN at a marginal rate that equals execution probability $\pi < 0.7$. Expected returns on the DM rise at a marginal rate of 1. If execution probability is not affected, an increase in x lowers the expected payoff from going to the CN instead of the DM at a rate $1 - \pi$. On the other hand, an increase in the switching point up to which traders use the CN may change the probabilities of order execution in a way that depends on the assumed probability distributions. This effect may be positive and may even exceed the negative partial derivative. Hence, the net effect depends on the probability distributions, as does multiplicity of equilibria. For a uniform distribution of values of trade and signals the latter effect vanishes, because agents always attribute the same probability to other signals being higher or lower than their own. The same holds for any other distribution if the variance of u^i is sufficiently small (Frankel, Morris and Pauzner, 2003).

Theorem 4 *Given a uniform distribution of values of trade and signals, there is a unique signal x^*, such that any equilibrium strategy assigns the CN to signals below x^* and the DM to signals above x^*. x^* is given by the solution to*

$$\tilde{U}(x^*, I_{x^*}) = \int_0^1 (x^* + \epsilon - 2\,\epsilon\,\alpha - t_{CN})\,\tilde{\pi}(\alpha)\,\mathrm{d}\alpha - x^* + t_{DM} = 0. \tag{24}$$

Given uniform distributions, there is a critical signal x^*, such that agents, who estimate the value of immediate trade to be lower than x^*, submit orders to the CN and agents with higher estimates go to the DM. Traders weigh expected gains from trading in the CN against certain gains from trading on the DM. At the equilibrium switching signal x^*, expected gains at both markets are equal. A trader i who receives signal x^* attaches equal probability to all values of θ within an $\epsilon-$surrounding of her own signal. If all traders who receive signals lower than x^* choose the CN, the proportion of traders in the CN is $\alpha(\theta, I_{x^*}) = \frac{x^*-\theta+\epsilon}{2\epsilon} \in [0,1]$ and the execution probability is $\pi^* = E(\pi(\theta, I_{x^*}) \mid x^*)$. For a marginal trader with $x^i = x^*$ the expected gain in the CN $\int_{x^*-\epsilon}^{x^*+\epsilon} (\theta - t_{CN})\,\pi(\theta, I_{x^*})\,\mathrm{d}\theta$ just compensates the gain in the DM $x^* - t_{DM}$ (see Figure 4).

Interpreting θ as a measure of price volatility, the theorem tells us that assets with a high price volatility are more likely to be traded on a DM, while assets, where the immediacy of order execution is less important, are traded in a CN. The higher x^*, the more likely is an asset traded in the CN.

It is worth noting that for uniform distributions the probability of order execution for the marginal trader is independent from the precision of signals.

Theorem 5 *With uniform distribution of values of trade and signals, the probability of order execution at the critical signal x^* is independent of ϵ and given by*

$$\pi^* = \int_0^1 \tilde{\pi}(\alpha)\,\mathrm{d}\alpha.$$

This result is, however, not robust with respect to changes in the probability distribution. Uniqueness of the equilibrium allows us to do some comparative statics:

Corollary 1 *Given a uniform distribution of θ, the critical signal x^* rises with rising λ or t_{DM} and with falling t_{CN} for any $\epsilon > 0$. If, in addition, the number of traders has a geometric distribution, a rise in ϵ lowers x^*.*

If we interpret the probability distribution of θ as statistical distribution of these values across all assets, the market share of the CN across all assets is prob$(x^i < x^*) = \frac{x^* - \check{\theta}}{\hat{\theta} - \check{\theta}}$. It rises in the difference of transaction costs between the two market venues and in market thickness λ. The latter tells us that CNs get especially those assets to trade, for which the expected turnover is large. But, even for blue chips with the highest turnover, the disutility of order execution (and hence, price volatility) is an important criterion for the allocation of initial orders.

Corollary 2 *Given a uniform distribution of values of trade and signals and geometric distribution of market size, for $\lambda \to \infty$, the critical signal x^* and θ_0 are both converging to $\frac{t_{DM} - t_{CN} \ln 2}{1 - \ln 2}$.*

As we have argued above, the probability of order execution is bounded below 1 even for an arbitrarily large number of expected traders. For increasing λ, x^* and θ_0 are both rising, and x^* is catching up with θ_0 for $\lambda \to \infty$. The solution of the private information game converges to the upper bound of the region with multiple equilibria in the common knowledge game. Although a large market does not guarantee order execution in the CN, it reduces strategic risk that the global–game solution accounts for. The global–game solution approaches an efficient allocation for $\lambda \to \infty$. Hence, for very large markets, private information is sufficient to establish efficient coordination.

On one hand, Corollary 2 tells us that even those assets with the largest turnover will not be traded in a CN if prices are too volatile. This property of the equilibrium limits the potentials for a CN to compete with a DM: if trading intervals are extended to collect a large number of orders (and raise λ), the probability of order execution rises, making the CN more attractive. But, the delay in the execution increases expected costs, which is a severe limitation to any CN.

If the distribution of θ is not uniform, there may be multiple equilibria. From Hellwig (2002) and Morris and Shin (2003) we know that uniqueness of a global game equilibrium is a quite general property, but requires that the dispersion of private information is sufficiently small. Frankel, Morris and Pauzner (2003) have shown that a large class of games with strategic complementarities has equilibria converging towards a single strategy profile that does not depend on higher moments of the probability distribution as the variance of private information approaches zero. Symmetric binary choice games with opposing dominance regions at the extrem ends of the space of a payoff relevant parameter belong to this class of games. For our game, these results imply that for any continuous distribution of random terms with vanishing variance of private information, the equilibrium is unique and converges to the equilibrium of the case with uniform distributions and vanishing ϵ.

Corollary 3 *Given a uniform distribution of values of trade and signals, as ϵ goes to zero, the critical signal x^* approaches*

$$x_0^* = \frac{t_{DM} - \pi^* \, t_{CN}}{1 - \pi^*}.$$

The limiting equilibrium x_0^* is an important reference point for equilibria under positive variance of private information. On the other hand, we know from Corollary 1 that for positive ϵ the equilibrium switching signal is smaller than x_0^*. This raises the question, how far the equilibrium may deviate from x_0^*.

Corollary 4 *Given a uniform distribution of values of trade and signals, for all $\epsilon > 0$,*

$$x_0^* - \frac{\pi^*}{1 - \pi^*} \, \epsilon < x^* < x_0^*.$$

This shows that precision of private information has no big impact on the critical signal x^*. Since $\frac{\pi^*}{1-\pi^*} < \frac{7}{3}$ for geometric distribution of market size, a reduction in ϵ raises x^* by a magnitude in the order of the change in ϵ. So even for positive ϵ, the equilibrium threshold is close to x_0^*. This is especially important as x_0^* is much easier to calculate than x^*. An example is illustrated in Figure 4.

The last result does also have some relevance for measures to compete for order flow. Corollary 1 has shown that a DM's unconditional market share increases in the precision of private information. But, according to Corollary 4 the provision of information, e.g. on price volatility, is a weak instrument to compete for order flow. The effect of increasing precision of private information is small and may easily be dominated by effects outside our model.

Insert Figure 4 here

Figure 4 *Nash equilibrium of the private information game. Agents switch markets at signal x^*, where the expected payoff from trading on the CN equals the certain payoff from trading on the DM, i.e. the areas A and B are of equal size. For $t_{CN} = 1$, $t_{DM} = 2$, $\lambda = 15$ and $\epsilon = 0.1$, we get $\theta_0 = 3.953$, $x^* = 3.434$ and $x_0^* = 3.445$.*

Given θ, the market share of the CN is $F(x^* \mid \theta)$, where F is the cumulative distribution of signals. If $\theta < x^* - \epsilon$, all traders get signals below x^* and choose to trade using the CN. If $\theta > x^* + \epsilon$, all traders get signals above x^* and trade on the DM. For $\theta \in [x^* - \epsilon, x^* + \epsilon]$ the market share of the CN is $\frac{x^* - \theta + \epsilon}{2\,\epsilon}$. As ϵ approaches zero, the CN's market share approaches 1 [0] for $\theta <$ [>] x_0^*.

Initial order flow is split between both markets if $\theta \in [x^* - \epsilon, x^* + \epsilon]$. However, this is an event with probability $\frac{2\,\epsilon}{\hat{\theta} - \check{\theta}}$. For small ϵ this probability is small and approaches zero for $\epsilon \to 0$. Hence, we would expect to observe a split of initial order flow only for very few assets. If the variance of private information approaches zero, all assets with $\theta > x_0^*$ (say assets with high price volatility) are exclusively traded on DMs while orders to trade assets with smaller θ are first submitted to the CN.

In comparison to the results of the common knowledge game, we see that uncertainty about the value of trade destabilizes a DM monopoly for low values of trade. Uncertainty makes it easier to coordinate trade on a CN, because traders believe that others might believe that some traders go to the CN anyway. Accordingly, there is a positive execution probability that is sufficient to attract some traders who know that the value of trade is above t_{DM}. For high values of trade up to θ_0, a CN could win the market provided that θ is common knowledge. Although this is a strong equilibrium, it is very hard to establish. Any grain of doubt that higher order beliefs of all traders coincide, leads to uncertainty which induces traders to submit initial orders to the DM, simply because they attach a positive probability to others behaving likewise. Heinemann, Nagel and Ockenfels (2004b) show that the best response to observed behavior in experiments on coordination games with multiple equilibria is close to the global–game solution.

In the common knowledge game, the CN–equilibrium provides higher expected payoffs to all traders whenever $\theta < \theta_0$. In the private information game, maximizing expected total payoffs to all traders requires to coordinate trade on the CN up to a signal k^* that is close to θ_0.

Theorem 6 *Given a uniform distribution of values of trade and signals, the strategy combination, maximizing expected total payoff to all players in the private information game is* I_{k^*}, *where*

$$k^* = \theta_0 - \epsilon \, \frac{\bar{\pi} - 2 \int_0^1 \alpha \, \tilde{\pi}(\alpha) \, \mathrm{d}\alpha}{1 - \bar{\pi}} < \theta_0. \tag{25}$$

Theorem 6 shows that the efficient strategy requires to switch markets at a signal that is smaller than θ_0. However, the deviation from θ_0 is smaller than $\frac{\bar{\pi}}{1-\bar{\pi}}\,\epsilon < \frac{7}{3}\,\epsilon$

and disappears for $\epsilon \to 0$, while x^* converges to $x_0^* < \theta_0$. So, for almost precise private information the equilibrium threshold is below the efficient one. For geometric distribution of market size, this property can be shown to hold for positive ϵ as well:

Theorem 7 *Given a uniform distribution of values of trade and signals and geometric distribution of market size, the unique equilibrium switching signal x^* is smaller than the collective payoff maximizing switching signal k^*.*

The equilibrium switching signal x^* is smaller than k^* and hence, the equilibrium is inefficient from traders' point of view. Reason are network externalities that arise from strategic complementarities. Agents should use the CN at signals at which, in equilibrium, they do not use it, because the decision to go to the CN increases expected payoff also for other users of the CN. The externality is not accounted for by individual decisions. This can be used to argue that CNs and other electronic market places need public support to overcome inefficiencies. It must be noted, however, that we assumed transaction fees at both markets to be set exogenously. In reality, costs to operate a CN are overhead and fees should be falling in the number of trades. Even more important, as orders carry information used for price discovery, a dealer's bid–ask spread is generally decreasing in trading volume. On one hand, this strengthens strategic complementarity that is the driving force to our results on the allocation of order flow. On the other hand, efficiency considerations must also consider that a CN can never replace a dealer market, because it free rides on the dealer's price discovery. A dealer, who gets only excess orders from a CN does also have to bear overhead costs and has less information than in the monopoly situation. This increases the bid–ask spread for the remaining orders and decreases prior expected transaction costs and efficiency.

For large markets, however, we find that that the equilibrium of the private information game approaches efficiency: assuming the geometric distribution for the number of traders, $\lambda \to \infty$ implies that $\tilde{\pi}(\alpha) \to \ln(2)$ for all $\alpha > 0$. Then, equation (25) implies that $k^* \to \theta_0$. Corollary 2 established that the equilibrium threshold converges to θ_0 as well. Thereby, even for positive ϵ the equilibrium converges to efficiency for $\lambda \to \infty$.

6 Robustness

In this section we check robustness of our results with respect to (i) combining private values with private information, (ii) endogenous trading costs, (iii) price discovery in the electronic market.

6.1 Private Values and Private Information

If utilities arising from immediate order execution differ across agents and the distribution of these private values is not common knowledge, we may get a unique equilibrium, even if all private values are in the intermediate region. Here, we only sketch a game with private values *and* private information.

Suppose that agent i has a private value of immediacy $\theta^i = \theta + y^i$. Let g be the distribution of y^i. Each trader knows her own preference θ^i. Density function g is common knowledge, but θ is unknown. Agents have a prior belief that θ has a uniform distribution on $[\check{\theta}, \hat{\theta}]$, and they get private signals x^i with a uniform posterior distribution in $[\theta - \epsilon, \theta + \epsilon]$.

This modelling approach is a generalization of the private value game and the private information game. If $y^i = 0$ with probability 1, we get the private information game with a common value. If $\epsilon = 0$, we are in the private value game with common knowledge of the distribution of private values. In the generalized approach, a strategy depends on θ^i and x^i. A symmetric equilibrium in monotone strategies is characterized by a decreasing function h, such that an agent chooses the CN if and only if $x^i < h(\theta^i)$. Such an equilibrium has the property that traders with low values and/or low signals about the distribution of private values choose the CN, while others go to the DM. Assets with large turnover or low θ are to a larger extend traded in the CN than assets with low turnover or high θ. The iterative elimination procedure now concerns h–functions, starting with $\bar{h}^0(\theta^i) = \hat{\theta} + \epsilon$ for all θ^i and $\underline{h}^0(\theta^i) = \check{\theta} - \epsilon$ for all θ^i. If the conditional variance of x^i given θ converges to zero, the equilibrium should be unique. A proof of this claim is, however, beyond the scope of this paper.

6.2 Endogenous Trading Costs

Trading costs depend on the market share. A CN must charge a fee covering the overhead costs of operating the network. It is a natural monopoly and average costs are decreasing in trading volume. For a DM, the bid ask spread can be lower, the more traders use this market. Reasons are, besides overhead costs, decreasing expected inventory per trade, lower costs of price discovery, and a lower risk of exploitation by insiders.

Suppose that t_{CN} is a decreasing function of the CN's market share α and t_{DM} is increasing in α. This strengthens strategic complementarity. If $t_{CN}(0) < t_{DM}(1)$, the lower dominance region exists. The assumption can be justified, by assuming that a CN cross–subsidizes dry markets by revenues from markets in which it is already established or reduces fees below average costs in the introduction phase as an advertisement.[2]

The DM will never charge an infinite fee, even if it has almost no customers. Recall that traded assets have a worth of their own and dealers may also use the CN to relieve themselves of involuntary inventory. Hence, the upper dominance region exists, provided that $\hat{\theta} > t_{DM}(0)$. Here, it is sufficient that some traders believe that other traders might believe ... that the DM is a dominant strategy for some traders. With the existence of both dominance regions strategic complementarity allows for the iterative elimination procedure leading to a unique equilibrium for sufficiently small noise in private information.

6.3 Price Discovery

Once a CN is established with a sufficient market share, it has an incentive to introduce its own price discovery by allowing for limit orders. Limit orders reduce the risk of an order not being executed to negligibility. With lower operating costs, an ECN appears being a dominant strategy for all traders, no matter how urgent their orders are. However, price discovery by market makers is more than just equalizing supply and demand. Typically, dealers try to smoothen prices over time by taking inventory. They also respond to indications of inside–trading. These services cannot be provided by an ECN. A low trading volume and uncertain prospects of the underlying asset are more likely leading to volatile prices in an ECN than in a DM. Hence, risk averse agents who have no clue about the fair price have an incentive to choose the DM for trading such assets. Traders, who know the fair price may submit limit orders to the CN but their orders might not being executed, if the current price happens to be beyond their limit, whereas prices are more stable at a DM, which raises the probability for these orders being executed.

No–arbitrage conditions limit the extend to which prices at a CN may deviate from prices at a DM to an amount in the order of $t_{CN} + t_{DM}$, where precise caps depend on the possible timing of arbitrage trades and on the volatility of the underlying fundamental.

Smoothing prices is a service that is valuable to customers as is immediate order execution. DMs provide both services, while pur electronic markets can provide

[2] With strategic price setting, the CN will set transactions fees below those of dealer markets if and only if the resulting equilibrium allocation of order flow guarantees a sufficient trade volume to cover overhead costs.

only one of the two. CNs provide smooth prices taken from the DM, ECNs provide almost certain order execution instead. For a trader, the trade–off between an ECN and a DM is similar to the trade–off between a CN and a DM. For traders with a low preference for the service of price–smoothing, the ECN is a dominant strategy, while trades with a high preference for smooth prices better choose the DM. Strategic complementarity arises from the fact that both market venues can reduce their price fluctuations with a higher turnover. Thus, both approaches, private value and private information game can be applied in a similar way to the competition between an ECN and DMs.[3]

7 Conclusion and Outlook

The proliferation of alternative trading systems such as CNs considerably increases inter–market competition for order flow. While increased competition may be welcomed from the perspective of market and price efficiency, the enhanced choice of trading venues fragments the order flow and reduces liquidity which is key to the functioning of financial markets. In this paper we presented a model building on the idea that the allocation of order flow between DMs and a CN may be understood as a coordination game among traders. The main advantage of a CN are low transaction costs. Part of the service provided by a dealer may be understood as an insurance against mismatch. Therefore, traders with a high preferences for fast order execution prefer trading on a DM even at higher costs.

The major achievement of our work is the removal of the multiplicity of equilibria in the allocation of order flow that has plagued all models in the previous literature. We prove existence of a unique equilibrium if traders have private information about the value of immediate order execution or if a sufficient mass of traders is patient enough to choose the CN as a dominant strategy.

In contrast to models with multiple equilibria, our analysis provides a definitive answer to the question, under which conditions a CN can co–exist with a dealer market. If traders have the same disutility from unexecuted orders, initial order flow concentrates on one market. Interpreting disutilities of unexecuted orders as a measure of price volatility, our model shows that orders to trade assets with low price volatility and large turnovers are initially submitted to a CN, while assets with

[3]This argument also explains an empirical difference between the US and Europe. In the US traditional markets operate as dealer markets and face competition by ECNs, while in Europe, most stock exchanges are organized as auctions and do not take inventory. They are not threatened by ECNs but face competition from cheaper CNs (Degryse and Van Achter, 2002).

high volatility or small volumes are exclusively traded on dealer markets. Empirical research has shown that, indeed, electronic trading systems are most successful in attracting orders to trade assets with high trading volume and low price volatility.[4] If disutilities differ sufficiently across individuals, both markets co–exist and initial order flow is fragmented. Traders with low disutilities use the CN and traders with higher disutilities go to the DM.

In Europe, CNs have been established mainly by brokers, who circumvent traditional exchanges by matching their customers' orders in–house, if possible. They use traditional exchanges only for excess orders. Although European exchanges are organized as auction systems, the features characterizing the dealer in our model fully applies to them. In the US, traditional dealer markets are less automated and price discovery takes explicit account of potential orders by insiders. Here, electronic communication networks (ECNs) have been particularly successful in attracting trade. ECNs combine direct trading with an automated auction system. While CNs free ride on the price discovery provided by traditional dealers and therefore require their co–existence, ECNs can completely replace traditional markets. However, the allocation of order flow follows a similar pattern: while bid–ask spreads of traditional dealers provide a partial insurance against exploitation of liquidity traders by insiders, transaction prices at ECNs do not provide this service.[5] Liquidity traders may find it worthwhile buying this insurance or not. Small differences in their probability assessments for insiders distorting prices lead to a unique equilibrium, in which all assets for which insiders are likely to be influential are exclusively traded on dealer markets, while other assets are exclusively traded on ECNs.

Electronic crossing networks have also emerged in product markets. Here, competition between CNs and intermediaries follows the same rules as in the game analyzed in this paper. CNs save transaction costs, but intermediaries provide reliable information on the products, smoothen prices or take the solvency and delivery risk that traders may face on electronic matching markets. For customers who attribute a high value to these services, it is dominant strategy to trade with intermediaries, while others, for whom the cost advantage is the major concern will trade on CNs. With two dominance regions, the results of our model carry over to these markets. Without dominance regions, there are multiple equilibria that make it difficult for a new trading venue to enter the market. However, by offering special services or a cost advantage that make a new market venue attractive for some customers even without liquidity, the new venue creates the minimal liquidity required to attract further customers and gain a substantial market share.

[4]For a recent empirical study see Theissen (2002).

[5]Empirical Evidence for this view is provided by Venkataraman (2001).

The games presented above provide a broad platform to study inter–market competition for order flow between DMs and a CN. With respect to further research, we can think of a number of extensions. Most importantly, one should make spread and transaction fees endogenous by considering inter–market competition of a dealer and a CN who set their prices strategically. If a dealer quotes her prices based on her expectations about the number of traders, market microstructure theory suggests that the spread must widen if the expected number of traders at the DM decreases to ensure coverage of the dealer's costs. On the other hand, Hendershott and Mendelson (2000) have shown that the presence of informed traders might lead to a reverse effect, because insiders prefer to hide their positions by submitting orders to a CN and, thus, reduces adverse selection at the DM.

In a dynamic, multiple–period model, a CN may set its fee strategically, even with a negative value, i.e. it offers little presents or bonus points to attract traders and gain market share in the beginning. Such modes of attracting liquidity were quite common in the early days of the internet economy. On the other side, a strategic dealer may lower prices for some periods in order to re–attract trade or to fend off competitors.

Another important extension of our model is the implementation of an endogenous price discovery process as it exists at ECNs. In this case, the price at which orders are executed is not taken from a primary exchange but is derived endogenously, depending on traders' preferences. In a continuous–trading model, buy and sell limit orders are submitted to the ECN and matched. Unexecuted orders are stored in the order book and matched against new incoming limit orders. This requires a dynamic model that discounts the expected utility from order execution in later periods.

Furthermore, our model can be used for analyzing the choice between direct marketing, search for trading partners or barter on one side and the use of intermediaries or media of exchange on the other side. Models explaining the use of money are closely related to the model employed above and typically suffer from multiple equilibria. We believe it being a promising task of future research to apply the global game approach in monetary theory.

Appendix

Proof of Lemma 1

Suppose the probability of a buyer [seller] to get a signal leading her to go to market

B is α_b $[\alpha_s]$. Then, the additional number of buyers k has a geometric distribution with $\mathrm{E}(k) = \alpha_b\,\lambda$. For a buyer on market B the probability of having k additional buyers on this market is

$$p_b(k) = \frac{1}{1+\alpha_b\,\lambda}\left(\frac{\alpha_b\,\lambda}{1+\alpha_b\,\lambda}\right)^k .$$

The probability of having r sellers is

$$p_s(r) = \frac{1}{1+\alpha_s\,\lambda}\left(\frac{\alpha_s\,\lambda}{1+\alpha_s\,\lambda}\right)^r .$$

The probability of execution of a buyer's order, given that there are k additional buyers and r sellers, is

$$\pi_b(k,r) = \begin{cases} \frac{r}{k+1} & \text{if} \quad r \le k \\ 1 & \text{if} \quad r > k. \end{cases}$$

The conditional probability of order execution, given that there are k additional buyers, is

$$\mathrm{E}(\pi_b\,|\,k) = \sum_{r=0}^{k} \frac{r}{k+1}\,p_s(r) + 1\,\cdot\,p_s(r>k) = \sum_{r=0}^{k} \frac{r}{k+1}\,p_s(r) + 1 - \sum_{r=0}^{k} p_s(r)$$

$$= 1 - \sum_{r=0}^{k}\left(1 - \frac{r}{k+1}\right)\,p_s(r) = 1 - \frac{1}{1+\alpha_s\,\lambda}\left[\sum_{r=0}^{k} q_s^r - \frac{1}{k+1}\sum_{r=0}^{k} r\,q_s^r\right],$$

where $q_s := \frac{\alpha_s\,\lambda}{1+\alpha_s\,\lambda}$. Using

$$1 - q_s = \frac{1}{1+\alpha_s\,\lambda}, \qquad \sum_{r=0}^{k} q^r = \frac{1-q^{k+1}}{1-q} \qquad \text{and} \qquad \sum_{r=0}^{k} r\,q^r = \frac{q\,[1-q^k\,(k+1-q\,k)]}{(1-q)^2}$$

we find that

$$\mathrm{E}(\pi_b\,|\,k) = q_s^{k+1} + \frac{\alpha_s\lambda}{k+1}\left[1 - q_s^k\,(k+1-q_s\,k)\right]$$

$$= q_s^{k+1} + \alpha_s\,\lambda\left[\frac{1-q_s^{k+1}}{k+1} - q_s^k + q_s^{k+1}\right] = q_s^{k+1}\,(1+\alpha_s\,\lambda) + \alpha_s\,\lambda\left[\frac{1-q_s^{k+1}}{k+1} - q_s^k\right].$$

The probability of order execution is

$$\mathrm{E}(\pi_b) = \sum_{k=0}^{\infty} \mathrm{E}(\pi_b \,|\, k)\, p_b(k)$$

$$= \sum_{k=0}^{\infty} \left[q_s^{k+1} \, (1 + \alpha_s \, \lambda) + \alpha_s \, \lambda \, \left(\frac{1 - q_s^{k+1}}{k+1} - q_s^k \right) \right] \frac{1}{1 + \alpha_b \, \lambda} \, q_b^k$$

where $q_b := \frac{\alpha_b \, \lambda}{1 + \alpha_b \, \lambda}$. This equals

$$\frac{1 + \alpha_s \, \lambda}{1 + \alpha_b \, \lambda} \sum_{k=0}^{\infty} q_s^{k+1} \, q_b^k + \frac{\alpha_s \, \lambda}{1 + \alpha_b \, \lambda} \sum_{k=0}^{\infty} \left(\frac{q_b^k}{k+1} - \frac{q_s^{k+1} \, q_b^k}{k+1} - q_s^k \, q_b^k \right)$$

$$= \frac{1 + \alpha_s \, \lambda}{1 + \alpha_b \, \lambda} \, q_s \sum_{k=0}^{\infty} q_s^k \, q_b^k + \frac{\alpha_s \, \lambda}{1 + \alpha_b \, \lambda} \left[\frac{1}{q_b} \sum_{k=1}^{\infty} \frac{q_b^k - q_s^k \, q_b^k}{k} - \sum_{k=0}^{\infty} q_s^k \, q_b^k \right] = \frac{\alpha_s}{\alpha_b} \sum_{k=1}^{\infty} \frac{q_b^k - q_s^k \, q_b^k}{k}.$$

Using

$$\sum_{k=1}^{\infty} q^k / k = -\ln(1 - q)$$

we find that

$$\mathrm{E}(\pi_b) = \frac{\alpha_s}{\alpha_b} \, [\ln(1 - q_b \, q_s) - \ln(1 - q_b)] = \frac{\alpha_s}{\alpha_b} \, \ln \frac{1 + (\alpha_s + \alpha_b) \, \lambda}{1 + \alpha_s \, \lambda} = \frac{\alpha_s}{\alpha_b} \, \ln \left(1 + \frac{\alpha_b \, \lambda}{1 + \alpha_s \, \lambda} \right).$$

Execution probability for sell orders is calculated accordingly by changing subscripts b and s. QED

Proof of Lemma 2

Let a^* be a Nash equilibrium. Suppose $\alpha_b(\theta, a^*) > \alpha_s(\theta, a^*)$ for some θ. Then $\Pi(\alpha_b, \alpha_s) < \Pi(\alpha_s, \alpha_b)$ and $\tilde{U}_b(\theta, a^*) < \tilde{U}_s(\theta, a^*)$. On the other hand, $\alpha_b(\theta, a^*) > \alpha_s(\theta, a^*)$ implies $\alpha_b(\theta, a^*) > 0$ and $\alpha_s(\theta, a^*) < 1$ and therefore $\tilde{U}_b(\theta, a^*) \geq 0$ and $\tilde{U}_s(\theta, a^*) \leq 0$. This contradicts the inequality above. Therefore, $\alpha_b(\theta, a^*) = \alpha_s(\theta, a^*)$ for all θ. QED

Proof of Theorem 1

A strategy combination a^* is a Nash equilibrium iff either

$$\tilde{U}(\theta, \alpha(\theta, a^*)) > 0 \quad \wedge \quad \alpha(\theta, a^*) = 1 \tag{26}$$

$$\text{or} \quad \tilde{U}(\theta, \alpha(\theta, a^*)) < 0 \quad \wedge \quad \alpha(\theta, a^*) = 0 \tag{27}$$

$$\text{or} \quad \tilde{U}(\theta, \alpha(\theta, a^*)) = 0 \tag{28}$$

If $t_{CN} < \theta < t_{DM}$, then $\tilde{U}(\theta, \alpha) > 0$ for all α. This excludes (27) and (28), while (26) holds. So, in equilibrium $\alpha(\theta, a^*) = 1$.

If $\theta = t_{DM}$, then $\tilde{U}(\theta, \alpha) \geq 0$ for all α. This excludes (27). (26) holds, and (28) is equivalent to $\tilde{\pi}(\alpha) = 0 \Leftrightarrow \alpha(\theta, a^*) = 0$. There are two equilibria with market shares of zero and one for the CN.

(27) holds for all $\theta > t_{DM}$. (26) requires $(\theta - t_{CN})\, \tilde{\pi}(1) > \theta - t_{DM}$, which is equivalent to $\theta < \theta_0$. (28) $\Leftrightarrow$ $(\theta - t_{CN})\, \tilde{\pi}(\alpha) = \theta - t_{DM} \Leftrightarrow \tilde{\pi}(\alpha) = \frac{\theta - t_{DM}}{\theta - t_{CN}}$. $\tilde{\pi}$ is a continuous and increasing function in α reaching from zero to $\bar{\pi}$. Hence, there is a unique solution $\tilde{\alpha}(\theta) = \tilde{\pi}^{-1}\left(\frac{\theta - t_{DM}}{\theta - t_{CN}}\right) \leq 1$ for all $\theta \leq \theta_0$. For $\theta > \theta_0$, there is no solution to (28) with $\alpha \in [0, 1]$. QED

Proof of Lemma 3

Let a^* be a Nash equilibrium of the private value game. $a^{i*}(b) = 1[0]$ if $\tilde{U}_b(\theta^i, a^*) > [<]\ 0$ and $a^{i*}(s) = 1[0]$ if $\tilde{U}_s(\theta^i, a^*) > [<]\ 0$. $\tilde{U}_b(\theta^i, a)$ and $\tilde{U}_s(\theta^i, a)$ are positive for $\theta^i < t_{DM}$ and negative for $\theta^i > \theta_0$ for all a. As $\tilde{U}_b$ and $\tilde{U}_s$ are strictly decreasing in θ^i, there are a $\theta^*_b(a^*)$ and a $\theta^*_s(a^*)$, such that $a^{i*}(b) = 1[0]$ if $\theta^i < [>]\, \theta^*_b(a^*)$ and $a^{i*}(s) = 1[0]$ if $\theta^i < [>]\, \theta^*_s(a^*)$.

Suppose $\theta^*_b(a) < \theta^*_s(a)$. Then $\alpha_b(a) < \alpha_s(a)$, and $\pi_b(a) > \pi_s(a)$. Then $\tilde{U}^i_b(a) > \tilde{U}^i_s(a)$ for all i. This implies $\theta^*_b(a) > \theta^*_s(a)$ which contradicts the inequality above. Hence, $\theta^*_b(a) = \theta^*_s(a)$. QED

Proof of Theorem 2

Since $\hat{U}(\theta)$ is continuous and positive for $\theta < t_{DM}$ and negative for $\theta > \theta_0$, it is sufficient to show that $\hat{U}$ is strictly decreasing in equilibrium.

$$\frac{d\hat{U}(\theta)}{d\theta} = \tilde{\pi}(F(\theta)) - 1 + (\theta - t_{CN})\, \frac{\partial \tilde{\pi}}{\partial \alpha}\, f(\theta). \tag{29}$$

For geometric distribution of the number of buyers and sellers and uniform distribution of private values,

$$\frac{d\hat{U}(\theta)}{d\theta} = \tilde{\pi}(\alpha) - 1 + \frac{\theta - t_{CN}}{\hat{\theta} - t_{CN}} \, \frac{\lambda}{(1 + 2\,\alpha\,\lambda)\,(1 + \alpha\,\lambda)}.$$

$\frac{\theta - t_{CN}}{\hat{\theta} - t_{CN}} = \alpha$ and in equilibrium $\tilde{\pi}(\alpha) = \frac{\theta - t_{DM}}{\theta - t_{CN}}$.

$$\left.\frac{d\hat{U}(\theta)}{d\theta}\right|_{\theta^*} = \frac{\theta - t_{DM}}{\theta - t_{CN}} - 1 + \frac{\alpha\,\lambda}{1 + 3\,\alpha\,\lambda + 2\,\alpha^2\,\lambda^2} = -\frac{t_{DM} - t_{CN}}{\theta - t_{CN}} + \frac{\alpha\,\lambda}{1 + 3\,\alpha\,\lambda + 2\,\alpha^2\,\lambda^2}.$$

$$\begin{aligned}
\left.\frac{d\hat{U}(\theta)}{d\theta}\right|_{\theta^*} < 0 \quad &\Leftrightarrow \quad \frac{t_{DM} - t_{CN}}{\hat{\theta} - t_{CN}} > \frac{\theta - t_{CN}}{\hat{\theta} - t_{CN}} \, \frac{\alpha\,\lambda}{1 + 3\,\alpha\,\lambda + 2\,\alpha^2\,\lambda^2} \\
&\Leftrightarrow \quad (1 + 3\,\alpha\,\lambda + 2\,\alpha^2\,\lambda^2)\,\frac{t_{DM} - t_{CN}}{\hat{\theta} - t_{CN}} > \alpha^2\,\lambda \\
&\Leftarrow \quad 2\,\alpha^2\,\lambda^2\,\frac{t_{DM} - t_{CN}}{\hat{\theta} - t_{CN}} \geq \alpha^2\,\lambda \quad \Leftrightarrow \quad \frac{t_{DM} - t_{CN}}{\hat{\theta} - t_{CN}}\,\lambda \geq \frac{1}{2}.
\end{aligned}$$

QED

Proof of Theorem 3

The efficient threshold θ^{**} maximizes (over k)

$$\mathrm{E}(U(\theta^i, a_k)) = \int_{t_{CN}}^{k} f(\theta)\,(\theta - t_{CN})\,\tilde{\pi}(F(k))\,\mathrm{d}\theta + \int_{k}^{\hat{\theta}} f(\theta)\,(\theta - t_{DM})\,\mathrm{d}\theta.$$

The first order condition implies

$$(\theta^{**} - t_{CN})\,\tilde{\pi}(F(\theta^{**})) + \int_{\check{\theta}}^{\theta^{**}} f(\theta)\,(\theta - t_{CN})\,\tilde{\pi}'(\cdot)\,\mathrm{d}\theta - \theta^{**} + t_{DM} = 0. \quad (30)$$

In equilibrium $\hat{U}(\theta^*) = 0$. If there is a unique equilibrium θ^*, the derivative of $\hat{U}$ is negative at equilibrium. Therefore, $\theta^* < \theta^{**}$ if and only if $\hat{U}(\theta^{**}) < 0$. (15) and (30) imply

$$\hat{U}(\theta^{**}) = (\theta^{**} - t_{CN})\,\tilde{\pi}(F(\theta^{**})) - \theta^{**} + t_{DM} = -\int_{\check{\theta}}^{\theta^{**}} f(\theta)\,(\theta - t_{CN})\,\tilde{\pi}'(\cdot)\,\mathrm{d}\theta < 0.$$

QED

Proof of Theorem 4

For uniform distribution of values of trade and signals,

$$\pi(\theta, I_x) = \begin{cases} \bar{\pi} & \text{if} \quad \theta < x - \epsilon \\ \tilde{\pi}\left(\frac{x-\theta+\epsilon}{2\,\epsilon}\right) & \text{if} \quad x - \epsilon \le \theta \le x + \epsilon \\ 0 & \text{if} \quad \theta > x + \epsilon. \end{cases}$$

and

$$\tilde{U}(x, I_x) = \frac{1}{2\,\epsilon} \int_{x-\epsilon}^{x+\epsilon} (\theta - t_{CN})\, \pi(\theta, I_x)\, \mathrm{d}\theta - x + t_{DM}.$$

Probability $\pi(\theta, I_x)$ depends on the difference between x and θ and the integral is evaluated around x. Substituting α for $\frac{x-\theta+\epsilon}{2\,\epsilon}$, we find

$$\tilde{U}(x, I_x) = \int_0^1 (x + \epsilon - 2\,\epsilon\,\alpha - t_{CN})\, \tilde{\pi}(\alpha)\, \mathrm{d}\alpha - x + t_{DM}$$

and

$$\frac{\mathrm{d}\,\tilde{U}}{\mathrm{d}\,x} = \int_0^1 \tilde{\pi}(\alpha)\, \mathrm{d}\alpha - 1 < 0.$$

As $\tilde{U}(x, I_x)$ is strictly decreasing in x, there is a unique x^* with $\tilde{U}(x^*, I_{x^*}) = 0$. QED

Proof of Theorem 5

For uniform distribution of values of trade and signals, the probability of order execution at the critical signal x^* is

$$\pi^* = \mathrm{E}(\pi(\theta, I_{x^*}) \,|\, x^*) = \frac{1}{2\,\epsilon} \int_{x^*-\epsilon}^{x^*+\epsilon} \tilde{\pi}\left(\frac{x^* - \theta + \epsilon}{2\,\epsilon}\right) \mathrm{d}\theta = \int_0^1 \tilde{\pi}(\alpha)\, \mathrm{d}\alpha.$$

QED

Proof of Corollary 1

From (24), the derivatives of x^* w.r.t. t_{DM}, t_{CN}, and λ are obvious. Differentiating (24) yields

$$\frac{\mathrm{d}x^*}{\mathrm{d}\epsilon} = \frac{\int_0^1 (1 - 2\,\alpha)\, \tilde{\pi}(\alpha)\, \mathrm{d}\alpha}{\int_0^1 (1 - \tilde{\pi}(\alpha))\, \mathrm{d}\alpha}.$$

As $\tilde{\pi}(\alpha) < 1$ for all $\alpha \in [0, 1]$, the denominator is positive. Splitting the numerator up, substituting y for $1 - \alpha$ in the second integral and rearranging terms gives

$$\int_0^1 (1 - 2\,\alpha)\,\tilde{\pi}(\alpha)\mathrm{d}\alpha = \int_0^{1/2} (1 - 2\,\alpha)\,\tilde{\pi}(\alpha)\mathrm{d}\alpha - \int_0^{1/2} (1 - 2\,y)\,\tilde{\pi}(1 - y)\,\mathrm{d}y$$

$$= \int_0^{1/2} (1 - 2\,\alpha)\,[\tilde{\pi}(\alpha) - \tilde{\pi}(1 - \alpha)]\,\mathrm{d}\alpha$$

For $\alpha < 1/2$, $\tilde{\pi}(\alpha) < \tilde{\pi}(1 - \alpha)$. Hence, the integral is negative. This proofs that x^* falls with rising ϵ. QED

Proof of Corollary 2

For $\lambda \to \infty$, the probability of order execution $\tilde{\pi}(\alpha)$ approaches $\ln 2$ for all $\alpha > 0$. Thus, equilibrium condition (24) converges to

$$\int_0^1 (x^* + \epsilon - 2\,\epsilon\,\alpha - t_{CN})\,\ln 2\,\mathrm{d}\alpha - x^* + t_{DM} = 0 \quad \Leftrightarrow \quad x^* = \frac{t_{DM} - t_{CN}\,\ln 2}{1 - \ln 2}.$$

Since $\bar{\pi} \to \ln 2$, θ_0 converges to the same value as x^*. QED

Proof of Corollary 3

As ϵ approaches zero, (24) shows that $\tilde{U}(x^*, I_{x^*}) \to (x^* - t_{CN})\,\pi^* - x^* + t_{DM} = 0$. Solving for x^* gives the equation in Corollary 3. QED

Proof of Corollary 4

Corollaries 1 and 3 imply $x^* < x_0^*$. From Theorem 4 we know that $\tilde{U}(x^*, I_{x^*}) = 0$.

$$\tilde{U}(x^*, I_{x^*}) = (x^* - \epsilon - t_{CN})\,\pi^* + 2\,\epsilon \int_0^1 (1 - \alpha)\,\tilde{\pi}(\alpha)\,\mathrm{d}\alpha - x^* + t_{DM} = 0.$$

As the integral is positive,

$$(x^* - \epsilon - t_{CN})\,\pi^* - x^* + t_{DM} < 0 \quad \Leftrightarrow \quad x^* > x_0^* - \epsilon\,\frac{\pi^*}{1 - \pi^*}.$$

QED

Proof of Theorem 6

Unconditional expected payoff to traders who employ switching strategy I_k is

$$\mathrm{E}(U(I_k)) = \frac{1}{\hat\theta - \breve\theta} \int_{\breve\theta}^{\hat\theta} F(k|\theta)\,(\theta - t_{CN})\,\tilde\pi(F(k|\theta)) + (1 - F(k|\theta))\,(\theta - t_{DM})\,\mathrm{d}\theta.$$

$$= \frac{1}{\hat\theta - \breve\theta}\left[\int_{\breve\theta}^{k-\epsilon} (\theta - t_{CN})\,\bar\pi\,\mathrm{d}\theta + \int_{k+\epsilon}^{\hat\theta} (\theta - t_{DM})\,\mathrm{d}\theta\right.$$

$$\left. + \int_{k-\epsilon}^{k+\epsilon} \frac{k-\theta+\epsilon}{2\,\epsilon}\,(\theta - t_{CN})\,\tilde\pi\left(\frac{k-\theta+\epsilon}{2\,\epsilon}\right) + \frac{\epsilon - k + \theta}{2\,\epsilon}\,(\theta - t_{DM})\,\mathrm{d}\theta\right]$$

$$= \frac{1}{\hat\theta - \breve\theta}\left[\int_{\breve\theta}^{k-\epsilon} (\theta - t_{CN})\,\bar\pi\,\mathrm{d}\theta + \int_{k+\epsilon}^{\hat\theta} (\theta - t_{DM})\,\mathrm{d}\theta\right.$$

$$\left. + 2\,\epsilon \int_0^1 \alpha\,(k + \epsilon - 2\,\epsilon\,\alpha - t_{CN})\,\tilde\pi(\alpha) + (1-\alpha)\,(k + \epsilon - 2\,\epsilon\,\alpha - t_{DM})\,\mathrm{d}\alpha,\right]$$

where we substituted α for $\frac{k-\theta+\epsilon}{2\,\epsilon}$. If there is an interior optimum k^*, the derivative $\mathrm{dE}(U(I_k))/\mathrm{d}k$ equals zero at k^*.

$$\frac{\mathrm{dE}(U(I_k))}{\mathrm{d}k} = \frac{1}{\hat\theta - \breve\theta}\left[(k - \epsilon - t_{CN})\,\bar\pi - k - \epsilon + t_{DM} + 2\,\epsilon \int_0^1 \alpha\,\tilde\pi(\alpha) + (1-\alpha)\,\mathrm{d}\alpha\right].$$

Setting the derivative to zero and solving for k gives (25). QED

Proof of Theorem 7

Given uniform distribution of values of trade and signals, the proof of Theorem 4 shows that $\tilde U(x, I_x)$ is strictly decreasing in x. At the equilibrium switching signal $\tilde U(x^*, I_{x^*}) = 0$. Hence, $x^* < k^*$ is equivalent to $\tilde U(k^*, I_{k^*}) < 0$. Using (24), (25, and Theorem 5) we find

$$\tilde U(k^*, I_{k^*}) = \int_0^1 \left[\theta_0 - \epsilon\,\frac{\bar\pi - 2\int_0^1 \alpha\,\tilde\pi(\alpha)\,\mathrm{d}\alpha}{1 - \bar\pi} + \epsilon(1 - 2\,\alpha) - t_{CN}\right] \tilde\pi(\alpha)\,\mathrm{d}\alpha$$

$$-\theta_0 + \epsilon\,\frac{\bar\pi - 2\int_0^1 \alpha\,\tilde\pi(\alpha)\,\mathrm{d}\alpha}{1 - \bar\pi} + t_{DM}$$

$$= \theta_0 (\pi^* - 1) - \pi^* t_{CN} + t_{DM} + \frac{\epsilon \Delta}{1 - \bar{\pi}}$$

where $\Delta = \pi^* (1 - 2\bar{\pi}) + \bar{\pi} - 2(2 - \pi^* - \bar{\pi}) \int_0^1 \alpha \tilde{\pi}(\alpha) \, \mathrm{d}\alpha$. Henceforth, $x^* < k^*$ is equivalent to

$$\frac{\epsilon \Delta}{1 - \bar{\pi}} < \theta_0 (1 - \pi^*) - t_{DM} + \pi^* t_{CN}$$

$$\Leftrightarrow \quad \epsilon \Delta < (t_{DM} - \bar{\pi} t_{CN})(1 - \pi^*) - (1 - \bar{\pi})(t_{DM} - \pi^* t_{CN}) = (t_{DM} - t_{CN})(\bar{\pi} - \pi^*)$$

Given our assumption that $\epsilon < t_{DM} - t_{CN}$, a sufficient condition for this is

$$\Delta < \bar{\pi} - \pi^*$$

$$\Leftrightarrow \quad (1 - \bar{\pi}) \pi^* < (2 - \pi^* - \bar{\pi}) \int_0^1 \alpha \tilde{\pi}(\alpha) \, \mathrm{d}\alpha.$$

$$\Leftarrow \quad 2(1 - \bar{\pi}) < 2 - \pi^* - \bar{\pi} \qquad \wedge \qquad \frac{\pi^*}{2} < \int_0^1 \alpha \tilde{\pi}(\alpha) \, \mathrm{d}\alpha.$$

$$\Leftrightarrow \quad -\bar{\pi} < -\pi^* \qquad \wedge \qquad \int_0^1 \tilde{\pi}(\alpha) \, \mathrm{d}\alpha < 2 \int_0^1 \alpha \tilde{\pi}(\alpha) \, \mathrm{d}\alpha.$$

$$\Leftrightarrow \quad \bar{\pi} > \pi^* \qquad \wedge \qquad \int_0^1 (1 - 2\alpha) \tilde{\pi}(\alpha) \, \mathrm{d}\alpha < 0.$$

These inequalities follow from monotonicity of $\tilde{\pi}$. The second has been proved above in Corollary 1. QED

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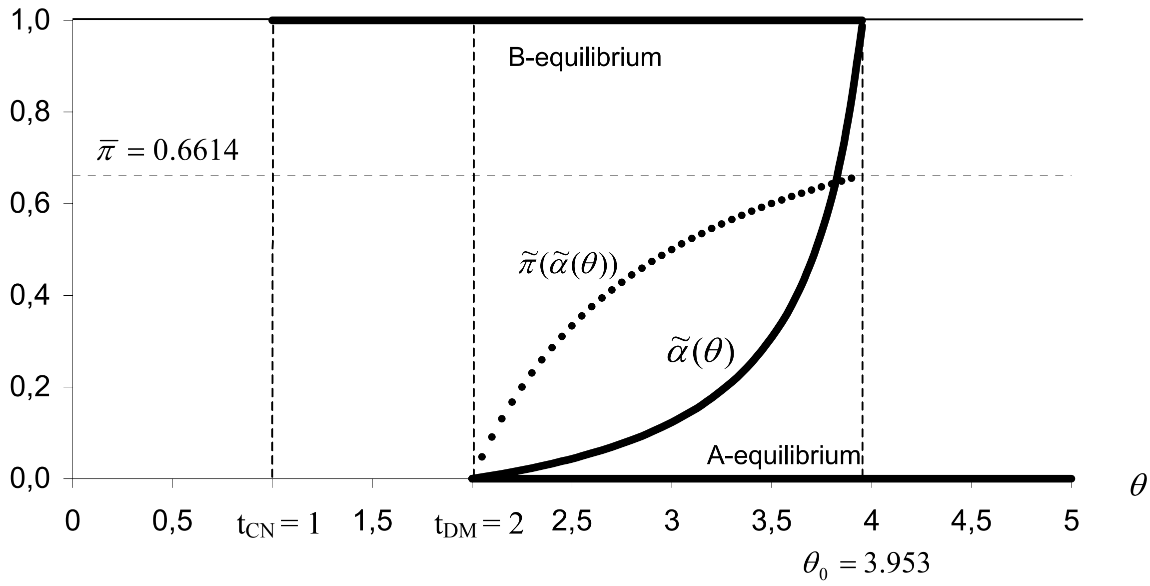

Figure 1. Nash equilibria of the common knowledge game. For $t_{CN} = 1$, $t_{DM} = 2$ and $\lambda = 15$, the upper limit of the multiplicity region is $\theta_0 = 3.953$, while $\bar{\pi} = 0.6614$.

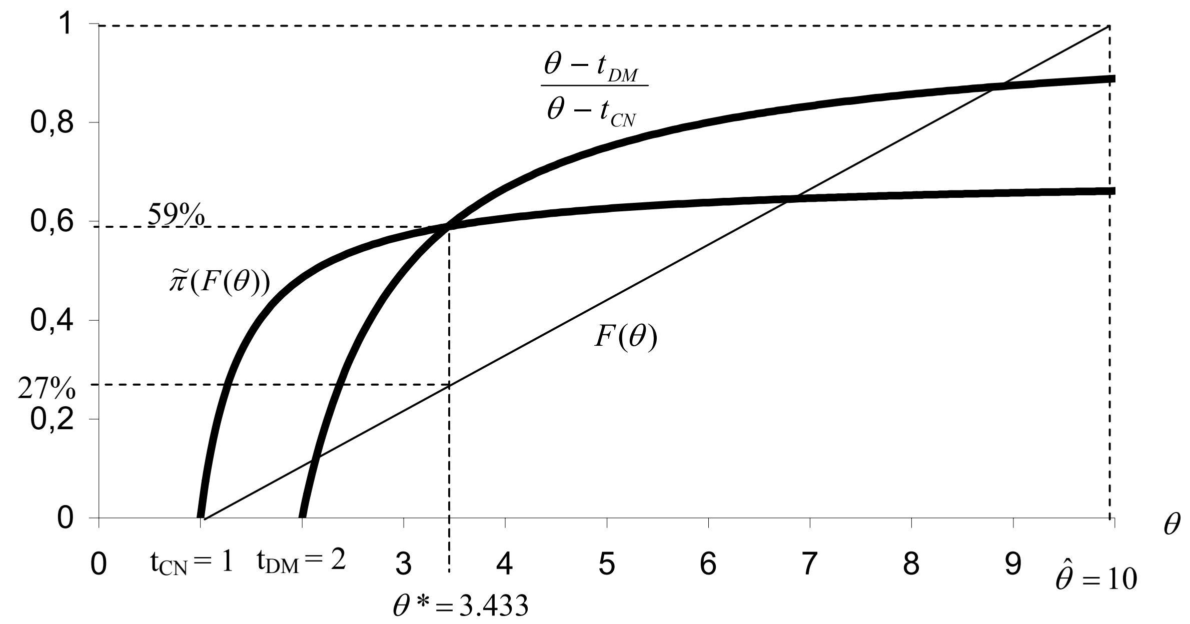

Figure 2. Nash equilibrium of the private value game. An intersection of the two curves at θ^* represents a Nash equilibrium if all agents with private values below θ^* go to B and agents with private values above θ^* go to A. For $t_{CN} = 1$, $t_{DM} = 2$, $\lambda = 15$ and $\hat{\theta} = 10$, we get $\theta_0 = 3.953$ and $\theta^* = 3.433$. The share of initial order flow submitted to the CN is F(θ^*) = 27%. Execution probability is $\tilde{\pi}(F(\theta^*) = 59\%$.

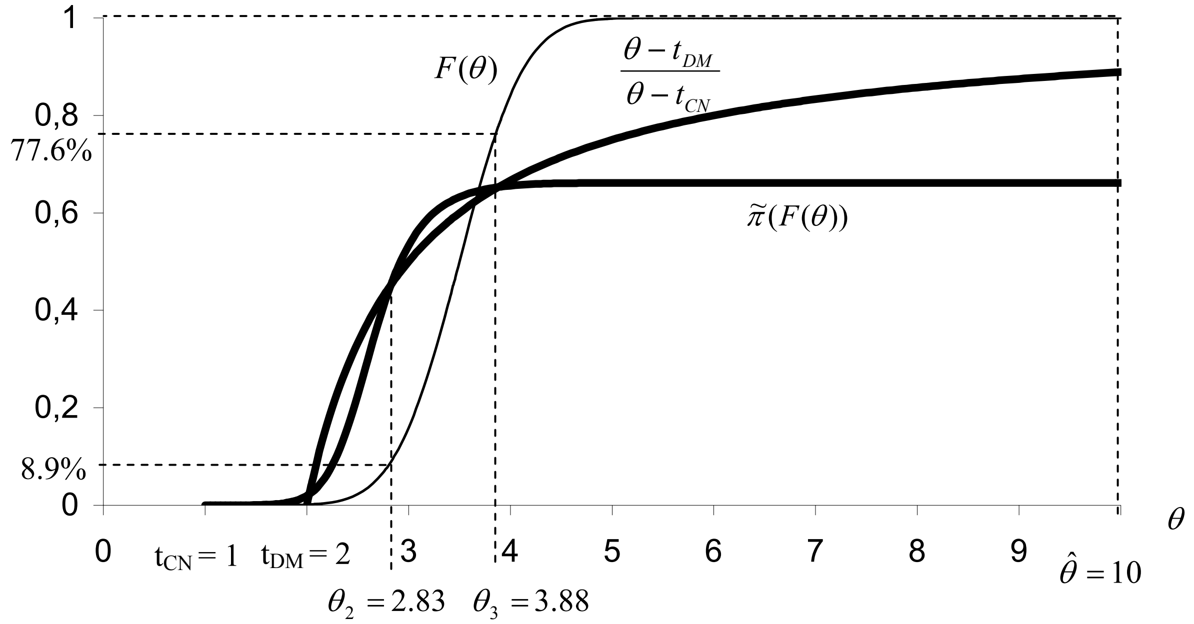

Figure 3. Multiple Nash equilibria in the private value game with a truncated normal distribution of private values, where E(θ) = 3.5 and V(θ) = 0.25. Truncation points are $\breve{\theta} = 1$ and $\hat{\theta} = 10$. There are three equilibria at $\theta_1 = 2.02$, $\theta_2 = 2.83$ and $\theta_3 = 3.88$. Associated shares of initial order flow submitted to the CN are 0.2%, 8.9% and 77.6%.

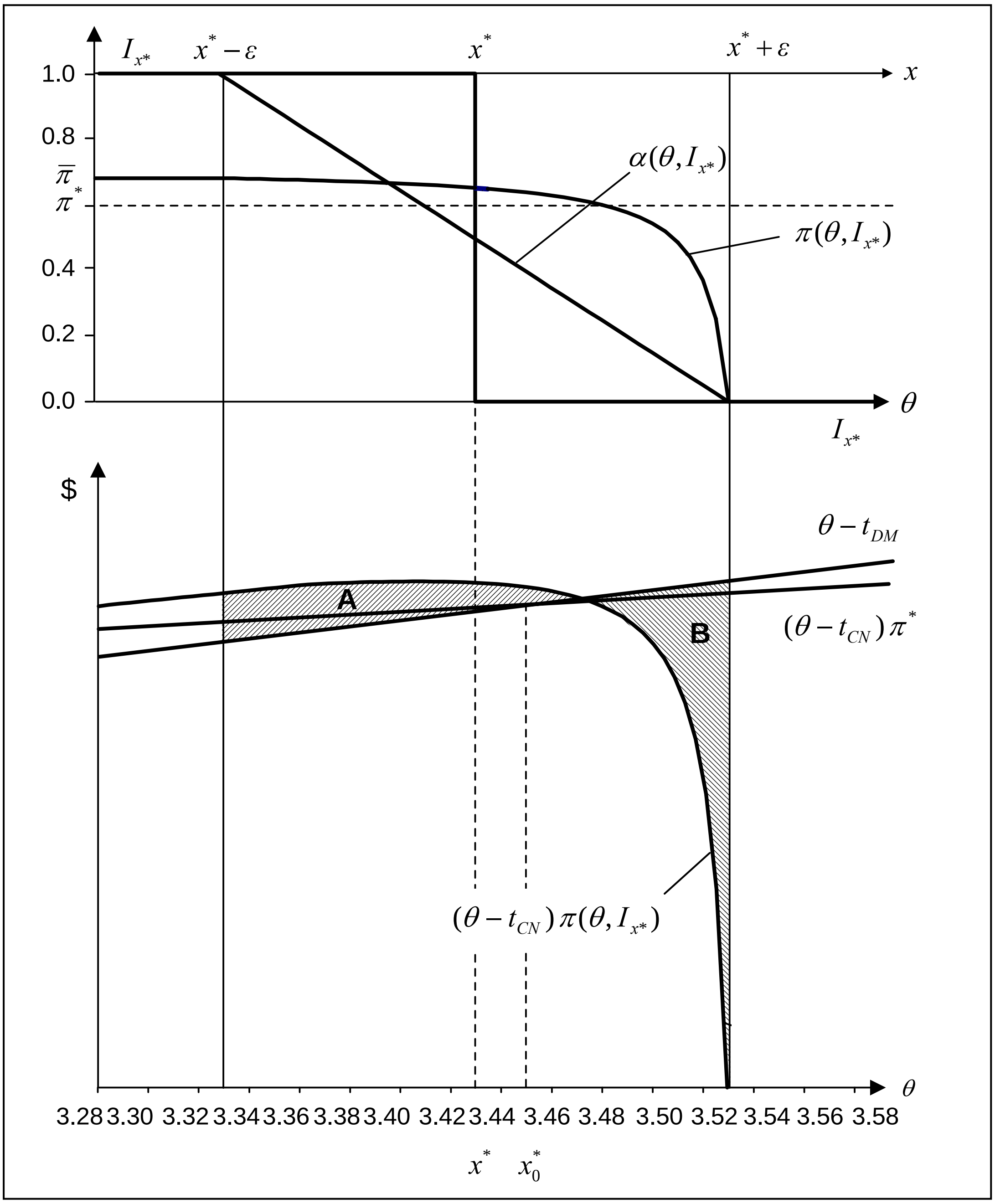

Figure 4. Nash equilibrium of the private information game. Agents switch markets at signal x^*, where the expected payoff from trading on the CN equals the certain payoff from trading on the DM, i.e. the areas A and B are of equal size. For $t_{CN} = 1$, $t_{DM} = 2$, $\lambda = 15$ and $\varepsilon = 0.1$, $\theta_0 = 3.953$, $x^* = 3.434$ and $x_0^* = 3.445$.