WAGE INDEXATION: A MACROECONOMIC APPROACH

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This essay examines the role of wage indexation in dampening macroeconomic fluctuations in a simple neoclassical model modified to incorporate short-term wage rigidities and uncertainty. The analysis departs from most of the previous literature on indexing in its explicit consideration of real disturbances. It is found that while indexing insulates the real sector from the effects of monetary shocks, it may exacerbate the real effects of real shocks. Thus the analysis suggests an optimal degree of partial indexation that depends on the underlying stochastic structure of the economy. Consequently, optimal indexing will not, in general, insulate the real sector from monetary variability.

1. Introduction

This paper develops a framework for investigating the role of indexation in dampening macroeconomic fluctuations. The distinguishing feature of the analysis is the emphasis on real as well as monetary disturbances as a source of price and output variation. It is found that while indexing insulates the real sector from the effects of monetary shocks, it may exacerbate the real effects of real shocks. For an economy subject to both types of disturbances, this result argues against the usual prescription of full indexation as a cure for the ills of monetary uncertainty. Rather, the analysis suggests an optimal degree of partial indexation that depends on the underlying stochastic structure of the economy. Among the conclusions of the paper are the following: the incentive to index is related to the variability of the price level, not to its mean rate of change. The optimal degree of indexation is partial, not full. Indexing will not, in general, completely neutralize monetary variability; thus policies that bring about increased monetary uncertainty impose unavoidable costs on the economy.

The design of the paper is to develop, in the second section, a simple stochastic macroeconomic model that is subsequently subjected, in section 3, to several

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1 The stabilization aspects of the indexing issue were recognized by a variety of early writers including Jevons (1896), Marshall (1925), Fisher (1922) and Pigou (1933), as well as later proponents of indexing such as Friedman (1974). However, with the exception of Pigou (1933, p. 295), these writers confine their discussions to frameworks which, either implicitly or explicitly, admit only monetary disturbances.
experiments that illustrate the impact of indexing on the size of macroeconomic fluctuations. In section 4 an optimal degree of indexing is derived and in section 5 the cost of increased monetary variability is explored. A summary of the conclusions and implications of the paper is contained in the final section.

2. The model

The discrete time framework described in this section is a simple neoclassical model modified to incorporate short-term wage rigidities and uncertainty. The wage rigidities are produced by a contracting scheme that calls for the setting of a nominal base wage rate and an indexing parameter before full information on the economic variables relevant to production decisions is received. Uncertainty is incorporated in the form of stochastic elements in the money supply and production functions that generate, respectively, monetary and real shocks to the system. Since the base nominal wage is fixed, these shocks may cause employment and output fluctuations through changes in the real wage rate (via price level fluctuations) and in the marginal product of labor. The importance of the indexing parameter stems from its role in determining the sensitivity of the real wage rate to price level changes. If the indexing parameter is set at zero, no adjustment in the negotiated nominal base wage is made for changes in the price level, and in consequence the real wage rate varies inversely with prices. If, on the other hand, the indexing parameter is set at one, the base wage is fully adjusted for changes in purchasing power and the real wage rate remains constant in the face of price level fluctuations. As is demonstrated in later sections, the desirability of a real wage rate that is unresponsive to price level variations—and therefore the attractiveness of indexing—depends on whether the variations are generated by real or monetary disturbances.

The paper is concerned with the distinction between real and nominal aggregate shocks. Accordingly, relative price and quantity fluctuations are excluded by specifying a one commodity model. Because indexing is designed to provide protection from unexpected movements in the price level, the model abstracts from anticipated changes and trends by postulating stochastic disturbances with zero mean. The capital stock is assumed fixed, so aggregate output

\[ \text{The indexing parameter (}\gamma\text{) is formally defined in section 4. For the purposes of the present section, it is sufficient to note that the size of the indexing parameter determines the extent to which nominal wage rates are adjusted for changes in the price level.} \]

\[ \text{Fully anticipated movements in the price level are taken into consideration during contract negotiations and do not require the kind of ex post adjustment of the nominal wage rate for price level changes which indexing provides. Therefore, in an economy experiencing a perfectly anticipated constant rate of inflation, there is no case for indexation, regardless of the level of that rate. However, the greater the uncertainty associated with any mean rate of inflation, the stronger the case for Indexing. Thus the incentive to index is related to the (imperfectly anticipated) variability of the price level, not to its mean rate of change.} \]
can be written as a function of total labor input and a productivity factor, \( \alpha \):\(^4\)

\[
Y_t = \alpha_t G(L_t), \quad \alpha_t = 1 + \mu_t. \tag{1}
\]

\( G(L_t) \) is homogeneous of degree less than one and \( \mu_t \) is a symmetrically distributed stochastic term with zero mean.

The nominal money supply is generated by the stochastic process,

\[
M_t^S = \beta_t M, \quad \beta_t = 1 + \xi_t, \tag{2}
\]

where \( M \) is constant and \( \xi_t \) is a symmetrically distributed stochastic term with zero mean. Note that \( \xi_t \) and \( \mu_t \) are uncorrelated. The demand for nominal money balances is assumed, for simplicity, to take the Cambridge form:

\[
M_t^D = kP_t Y_t, \tag{3}
\]

where \( k \), the desired ratio of money to nominal income, is constant. Consequently, the analysis does not consider stochastic shifts in velocity. Prices are assumed to adjust instantaneously to insure equilibrium in the money market,\(^5\)

\[
M_t^S = M_t^D. \tag{4}
\]

The level of employment is contingent on the nature of contracts and production as well as labor market conditions. Production is assumed to be a discrete process that takes place once each period. All contracts have a duration of one period and establish a base nominal wage rate \( (W^*) \) and an indexing parameter \( (\gamma) \). Contracts for any period \( t \) are written at the end of period \( t - 1 \), so these two control variables must be set with less than full information on the current variables relevant to production decisions in period \( t \). A number of potential complications are sidestepped by assuming that the base wage is set at the level that corresponds to equilibrium in the labor market when the realized values of the two disturbance terms are zero.\(^6\) This level is designated by an asterisk

\(^4\)The framework specified in this section can be viewed as an approximation of the log-linear system implemented in section 4 of the paper. In particular, the disturbance terms \( \mu \) and \( \xi \) of section 4 correspond to the logged values of the terms \( \alpha \) and \( \beta \) of the present section.

\(^5\)Using Walras’ Law, the goods market has been eliminated from the system. Explicit incorporation of this market would involve an aggregate excess demand function of the form

\[
EDY = Y_t^D - Y_t^S = H(M_t^S/P_t - M_t^D/P_t),
\]

where \( H \) is an increasing function of the state of excess supply in the money market and \( H(O^\gamma) = 0 \). Under this specification, equilibrium in the money market automatically implies equilibrium in the goods market.

\(^6\)There is no reason to assume that the wage rate negotiated by utility-maximizing economic agents would be the certainty equivalent specified here. The concern of the paper, however, is with the indexing parameter, not the wage rate, as a control variable. Accordingly, the problem of specifying a wage rule that approximates rational maximizing behavior is bypassed and an arbitrary wage setting mechanism is stipulated.
and is referred to as the certainty equivalent of the nominal wage rate.\textsuperscript{7} The labor market is described by eqs. (5) and (6). The supply and demand functions take familiar forms, with demand depending on the ratio of the real wage rate, \( w \), to the productivity factor, \( \alpha \):\textsuperscript{8}

\[
L^D_t = f\left(\frac{w_t}{\alpha_t}\right), \quad w_t = \frac{W_p}{P_t}, \quad f_{w/\alpha} < 0,
\]

\[
L^S_t = g(w_t), \quad g_w > 0.
\]  

Thus the base nominal wage rate can be found by equating labor demand to labor supply, subject to the constraint that the disturbances are at their means: \( \mu_t = \xi_t = 0 \).

The determinants of the indexing parameter are discussed in detail in section 4. For present purposes it can be noted that the value of the parameter must lie between zero and one, inclusively.

Once contracts have been negotiated, the values of all previously unknown stochastic terms associated with period \( t \) are realized and production decisions are made. At this point the level of employment becomes completely demand determined. Workers are assumed to supply whatever amount of labor is demanded by employers at the negotiated wage rate. In effect, once the wage rate is set, the supply of labor becomes perfectly elastic and eq. (6) is dropped from the system.\textsuperscript{9}

3. Response of the system to shocks

We turn now to an examination of some special cases designed to illustrate the impact of indexing on the size of macroeconomic fluctuations. It is shown that the effects of indexing depend crucially on whether price level fluctuations are generated by real or monetary disturbances. While indexing does prevent transmission of monetary shocks to the real sector, it also exacerbates the real effects of real disturbances.\textsuperscript{10} We begin by postulating an economy subject only

\textsuperscript{7}This concept should not be confused with the mathematical expectation of the nominal wage rate since, in general, the two concepts are not equivalent.

\textsuperscript{8}That \( w \) and \( \alpha \) enter the demand for labor function as a ratio can be seen by differentiating the production relationship with respect to labor and equating the result (the marginal product of labor) to the real wage rate.

\textsuperscript{9}The conclusions of the paper are robust with respect to variations in this assumption. The same qualitative results obtain if, for example, employment is determined by a 'short end' rule (employment being the lesser of the amounts supplied and demanded). The assumption on which the analysis does depend crucially is that to some extent levels of wages rather than levels of employment or income are negotiated.

\textsuperscript{10}Fischer (1975) arrives at similar conclusions using a framework broadly in accord with that of the present paper. For an alternative view of the nature of output and employment determination and, consequently, of the impact of indexing on macroeconomic fluctuations, see Barro (1975).
to monetary shocks. The behavior of the system is explored under two alternative indexing assumptions: no indexation and full indexation. The same set of experiments is subsequently performed on an economy subject to real disturbances.

3.1. Monetary shocks

In this section only monetary shocks to the system are considered. The possibility of real disturbances is eliminated by constraining the stochastic term in the production relationship to zero, leaving aggregate output exclusively a function of total labor input:\n
\[ Y = G(L). \]  

(1')

Likewise, labor demand in this case depends solely on the real wage rate:

\[ L^D = f(w). \]  

(5')

The response of the system to an unanticipated increase in the money supply can be analyzed using fig. 1, which provides a geometric interpretation of the impact of monetary shocks on the real sector. In quadrant I the dependence of the real wage rate on the disturbance term $\xi$ is represented. The nature of the dependence is contingent on the indexing assumption employed: in the case of an indexed economy the schedule is perfectly elastic, while for a nonindexed system it is negatively sloped. These results can be obtained from eqs. (2) and (3) and the assumption of continuous equilibrium in the money market. Setting nominal money supply equal to demand gives

\[ \beta \bar{M} = kPY, \quad \beta = 1 + \xi. \]  

(7)

Noting the definition of the real wage rate ($w = W/P$) and the exclusive dependence of output on the real wage (via the level of employment), eq. (7) can be rewritten in a more revealing form,

\[ (1 + \xi) \cdot \bar{M} = k \cdot (W/w) \cdot G[f(w)], \quad \bar{G}_w < 0. \]  

(8)

As $\xi$ increases, the nominal supply of money expands. Continued equilibrium in the money market requires a corresponding increase in the demand for nominal balances, as given by the right-hand side of eq. (8). This can be accomplished only by a fall in the real wage rate, a rise in the nominal wage rate, or both. It is at this point that the distinction between indexed and nonindexed systems

\[ 11 \text{Time subscripts are omitted here and in the following sections.} \]
becomes important. In a nonindexed economy, the indexing parameter is set at zero and no adjustment of the nominal wage rate is made for changes in the price level. Thus for a nonindexed system the nominal wage rate \( W \) of eq. (8) is a fixed parameter and the real wage rate \( w \) must fall in response to the postulated monetary shock. Hence, there is a negative relationship between \( w \) and \( \xi \) for \( \gamma = 0 \), as drawn in fig. 1. The intercept of this curve corresponds to the certainty equivalent of the real wage rate. In an indexed economy, on the other hand, the indexing parameter is set at unity and the nominal wage rate is fully adjusted for changes in the price level. Thus, the real wage rate is effectively fixed at its certainty equivalent, implying the perfectly elastic schedule depicted for \( \gamma = 1 \).

In the second quadrant the demand for labor is shown as a function of the real wage rate [eq. (5')] and in the third quadrant the production relationship [eq. (1')] is drawn.

Corresponding to \( \xi = 0 \), and denoted by asterisks, are the certainty equivalents of the real wage rate, employment, and output that characterize the economy at its mean. It is readily verified that for a nonindexed economy, an unanticipated increase in the money supply leads to levels of employment and output in excess of their certainty equivalents. Consider, for example, the effects of a positive monetary shock such as \( \xi \). The shock results in a real wage rate \( \bar{w} \) that falls short of the certainty equivalent of the real wage \( w^* \). This, in turn, induces levels of employment \( \bar{L}_n \) and output \( \bar{Y}_n \) that exceed their certainty equivalents \( L^* \) and \( Y^* \). Similarly, fig. 1 can be used to show that an unanticipated decrease in the money supply results in levels of output and employment that fall short of
Accordingly, monetary disturbances do have real effects in a non-indexed economy.

In an indexed economy the real wage rate is effectively fixed at its certainty equivalent, and, consequently, monetary disturbances have no real effects. Using fig. 1 again to trace the effects of a positive shock, it is apparent that the real wage rate \( \bar{w}_i \), employment \( \bar{L}_i \), and output \( \bar{Y}_i \) in the indexed economy are all equal to their certainty equivalents.

Clearly, then, the real sector is insulated from the effects of monetary shocks in a fully indexed economy. Failure to index, on the other hand, causes transmission of monetary disturbances to the real sector.

### 3.2. Real shocks

We turn now to the case of an economy subject only to real disturbances. For such an economy eq. (1) in its stochastic form is the relevant production relationship. Monetary shocks are excluded by constraining the disturbance term in the money supply function to zero, thereby fixing nominal balances at \( \bar{M} \).

\[
M^s = \bar{M}.
\]

The response of the system to real shocks can be demonstrated using diagrammatic apparatus similar to that of the preceding subsection. In the first quadrant of fig. 2 the dependence of the real wage rate on the real disturbance term \( \mu \) is depicted. Again, the nature of the relationship is determined by the indexing assumption employed; in the case of an indexed economy the schedule is perfectly elastic, while for a nonindexed system it is positively sloped. These results are obtained, as in the previous section, from the equilibrium condition for the money market,

\[
\bar{M} = k \cdot (W/w) \cdot (1 + \mu) \cdot G[f(\omega/(1+\mu))],
\]

where

\[
G_{w/a} = G_{w/(1+\mu)} < 0.
\]

With a fixed nominal supply of money, an increase in \( \mu \) must be offset by an increase in the real wage rate, a decrease in the nominal wage rate, or both. In a non-indexed economy the nominal wage rate is fixed, so the burden of adjustment falls on the real wage rate, which rises in response to increases in \( \mu \). This gives the positive relationship between \( w \) and \( \mu \) drawn for \( \gamma = 0 \) in fig. 2. In an indexed economy, the real wage rate is fixed at its certainty equivalent, implying the perfectly elastic schedule depicted for \( \gamma = 1 \).

In the second quadrant the demand for labor is again drawn as a function of real wage rate. In this subsection, however, labor demand depends on the
productivity factor, \( x = 1 + \mu \), as well as the real wage rate. Thus the increases in productivity associated with increases in \( \mu \) cause outward shifts in this schedule. Likewise, the position of the production relationship shown in the third quadrant is contingent on \( \mu \). Higher values of \( \mu \) correspond to greater levels of output at any given employment level, and therefore to downward shifts in the production schedule.

Using fig. 2 it can be shown that the effects on output and employment of a real shock are greater in an indexed economy than in a nonindexed economy. Consider, for example, a positive real shock such as \( \bar{\mu} \). In a nonindexed system the shock results in a real wage rate \( \bar{w}_n \) that exceeds the certainty equivalent \( w^* \). At the same time the labor demand schedule shifts outward, and the production function downward, relative to the schedules associated with \( \mu = 0 \). It is easily verified that the vertical shift of the labor demand schedule at \( L^* \) is equal to the increase in the real wage rate induced by the disturbance.\(^{12}\) Consequently, the

\[ \frac{d \log w}{d \log P} = \frac{d \log Y}{d \log P} = \frac{d \log A}{d \log P}. \]

At an employment level of \( L^* \), then, \( \frac{d \log w}{d \log MPL} = \frac{d \log MPL}{d \log MPL}. \]

\(^{12}\) The change in the real wage rate (relative to \( w^* \)) induced by \( \bar{\mu} \), holding employment constant at \( L^* \), is given by

\[ d \log w = d \log W - d \log P = d \log Y = \bar{\mu}. \]
increase in the wage rate is exactly offset by higher labor productivity, and employers have no incentive to alter the initial employment level of \( L^* \). At that level of employment, however, output \( (\bar{Y}_n) \) exceeds its certainty equivalent \( (\bar{Y}^*) \) due to the productivity increase associated with \( \bar{\mu} \). Similarly, fig. 2 can be used to show that an unanticipated decrease in productivity results in a level of output which falls short of \( Y^* \), while employment is maintained at \( L^* \). Accordingly, in a nonindexed economy, the impact of a real shock on output is limited by the response of the real wage rate, which adjusts to keep employment fixed at \( L^* \).

In an indexed economy the real wage rate is effectively fixed at \( w^* \) and, as a consequence, does not exert the moderating influence on output response demonstrated for the nonindexed case. Using fig. 2 again to trace the effects of a positive real shock, it is apparent that employment \( (\bar{L}_i) \) exceeds \( L^* \). Hence, the level of output in the indexed system exceeds both its certainty equivalent \( (Y^*) \) and the level achieved in the nonindexed system \( (\bar{Y}_n) \). Clearly, the real effects of real disturbances are more pronounced in an indexed economy than in a nonindexed economy.

The analysis of section 3 has demonstrated the differences in response of indexed and nonindexed economies to real and monetary shocks. The conclusions of the section are straightforward: while indexing is an effective means of insulating the real sector from monetary shocks, it exacerbates the real effects of real disturbances. At this point it seems intuitively appealing to conclude that in an economy subject to both real and monetary disturbances, there must exist a degree of indexing between zero and one which is in some sense optimal. As yet, however, no criterion has been established for the choice by rational decision-makers of an optimal degree of indexing. In particular, the extent to which it is desirable to moderate the real responses to real shocks has not yet been discussed. These issues are addressed in section 4, in which a welfare criterion based on deviations of actual output from a 'desired' level of output is proposed, and an optimal degree of indexing derived.

4. The optimal degree of indexation

In this part the formal structure of section 2 is extended to incorporate the remaining elements necessary to a discussion of the optimal degree of indexing. For computational ease, eqs. (1) through (6) are respecified in log linear form:

\[ \text{dlog}w = -\text{dlog}P = \text{dlog}Y = \text{dlog}x. \]

so that the ratio \( w/x \) is a constant. Consequently, real shocks have no effect on the level of employment in a nonindexed system.

\[ \text{This can also be seen from eq. (5), which specifies the demand for labor as a function of the ratio of the real wage rate to the productivity factor, } x. \]
\[ \log Y = \delta \log L + \mu, \quad (1') \]

\[ \log M^S = \log \bar{M} + \xi, \quad (2') \]

\[ \log M^D = \log k + \log P + \log Y, \quad (3') \]

\[ \log M^S = \log M^D, \quad (4') \]

\[ \log L^D = -\eta (\log w - \mu) + \eta \log \delta, \quad \eta = 1/(1 - \delta)^{14}, \quad (5') \]

\[ \log L^S = \omega \log w + \eta \log \delta. \quad (6') \]

The stochastic terms \( \mu \) and \( \xi \) are symmetrically distributed with mean zero. The elasticities of supply and demand for labor are given by \( \omega \) and \( \eta \), respectively, while \( \delta \) is the elasticity of real output with respect to labor input. The constant term in the labor supply function, \( \eta \log \delta \), is chosen so that \( \log w^* \) (the certainty equivalent of \( \log w \)) is equal to zero.

Risk neutral, rational decision-makers are postulated who correctly assess the underlying stochastic structure of the economy and, on the basis of that assessment, act to maximize social welfare. The welfare criterion is assumed to take the form of a loss function, \( Z \), which is expressed in terms of deviations of the log of actual output (log \( Y \)) from the log of a ‘desired’ level of output (log \( Y_o \)):

\[ Z = E[(\log Y - \log Y_o)^2]. \quad (10) \]

Desired output is defined as that level of output which would prevail in a perfectly frictionless version of the economy postulated in this paper. In such an economy, the log value of output would be given by

\[ \log Y_o = \delta \log L_o + \mu, \quad (11) \]

where \( L_o \) is the level of employment corresponding to the intersection of the labor supply and demand schedules given by eqs. (5') and (6'). The motivation for defining desired output in this way stems from the social inefficiency implied by points that lie off market supply and demand curves. Similarly, the particular

\[ ^{14} \text{The labor demand function is derived by differentiating the production relationship,} \]

\[ Y = e^{\gamma L^o}, \]

with respect to labor and equating the result (the marginal product of labor) to the real wage rate. This gives a demand function of the form

\[ L = (w/\delta e^\mu)^{1/\delta - 1}, \]

which, in log form, is eq. (5').
form that the loss function takes in eq. (10) is designed to approximate the loss in consumer's surplus associated with deviations from optimal output.

In the appendix the following expressions are developed for the logs of actual and optimal output:

\[
\begin{align*}
\log Y &= \delta \eta \left[ \frac{(1-\gamma)\xi + \gamma \mu}{1 + \delta \eta (1-\gamma)} \right] + \mu + \delta \eta \log \delta, \\
\log Y_o &= \delta \eta \mu[\omega/(\omega + \eta)] + \mu + \delta \eta \log \delta.
\end{align*}
\]

Subtracting (13) from (12), squaring the result, and taking expectations yields the following expression for the loss function,

\[
Z = \delta^2 \eta^2 \left\{ V_\mu \left[ \frac{\gamma}{1 + \delta \eta (1-\gamma)} - \frac{\omega}{\omega + \eta} \right]^2 + V_\xi \left[ \frac{(1-\gamma)}{1 + \delta \eta (1-\gamma)} \right]^2 \right\}.
\]

Here \( V_\mu \) and \( V_\xi \) are the variances of the disturbance terms \( \mu \) and \( \xi \), respectively.

Since welfare is maximized when the value of the loss function, \( Z \), is minimized, the optimal degree of indexation (denoted \( \gamma_o \)) can be obtained from the first-order conditions for a minimum. Differentiating eq. (14) with respect to \( \gamma \), setting the result equal to zero, and solving for \( \gamma_o \) gives

\[
\gamma_o = \theta + (1-\theta)\omega/(1+\omega),
\]

where

\[
\theta = \frac{V_\xi}{\left[ \frac{\eta^2 (1+\omega)}{\omega + \eta} \right]\mu + V_\xi}.
\]

Examination of the terms entering eq. (15) reveals that the optimal degree of indexing, \( \gamma_o \), is a weighted average of the optima corresponding to the extreme cases of \( V_\mu = 0 \) and \( V_\xi = 0 \). Setting \( V_\mu \) equal to zero results in a \( \gamma_o \) equal to one. This is the case discussed in section 3.1 of an economy subject only to monetary shocks. The rationale behind a policy of full indexation in such an economy is clear.

When, on the other hand, \( V_\xi \) is set equal to zero (the case of an economy subject only to real shocks), \( \gamma_o \) takes on the value \( \omega/(1+\omega) \), which is nonnegative and less than one. The rationale behind this result rests on the observation that some response of the real sector to real shocks is to be desired. Returning briefly to the framework of section 3, consider the case of a positive real shock such as \( \tilde{\mu} \) (fig. 3). The desired levels of output and employment, \( Y_o \) and \( L_o \), exceed those achieved in the nonindexed economy but fall short of those in the fully indexed economy. Clearly, the desired levels can be obtained through some
degree of indexing less than one but greater than zero. Within the specific framework of the present section, this optimal degree of indexing is independent of the elasticity of demand for labor ($\eta$) but is positively related to the elasticity of supply of labor ($\omega$). Manipulation of the slopes of the schedules drawn in fig. 3 provides the intuition behind these results.

For the general case in which the variances of both disturbance terms are nonzero, $\gamma_o$ lies between unity and $\omega/(1 + \omega)$. The dependence of $\gamma_o$ on the relative magnitudes of the two types of disturbances is reflected in the weights, $\theta$ and $(1 - \theta)$. As the relative size of monetary disturbances ($V_\theta$) increases, $\theta$ increases and $\gamma_o$ approaches unity. Conversely, as the relative size of real shocks ($V_\mu$) increases, $(1 - \theta)$ rises and $\gamma_o$ approaches $\omega/(1 + \omega)$. Much less obvious is the dependence of the optimal degree of indexing on the elasticities of supply and demand for labor. It can be shown, however, that $\gamma_o$ is positively related to $\omega$ and negatively related to $\eta$.

5. The costs of increased monetary variability

An important implication of models which incorporate monetary disturbances but which neglect real shocks is the following: in a freely adjusting, optimally indexed economy, increased monetary variability imposes no costs. In this section we show that for the more general case of a system subject to both real and monetary disturbances, the above proposition is not valid.
Formally, the costs of an increase in monetary variability can be shown by differentiating the loss function given by eq. (14) with respect to $V_{\xi}$, allowing optimal adjustment of indexing parameter to the increased monetary uncertainty,

$$\frac{dZ}{dV_{\xi}} = \frac{\partial Z}{\partial V_{\xi}} + (\frac{\partial Z}{\partial \gamma})(\frac{d\gamma}{dV_{\xi}})$$

$$= \delta^2 \eta^2 \left[ \frac{(1 - \gamma_o)}{1 + \eta \delta (1 - \gamma_o)} \right]^2$$

$$> 0. \quad (16)$$

As eq. (16) demonstrates, an increase in monetary variability unambiguously imposes costs, even in a freely adjusting, optimally indexed economy.

A less formal argument proceeds from the conclusions of the last section. It has already been shown that in a system subject to real as well as monetary shocks, the indexing parameter will not be set at unity. Full insulation of the real sector from monetary shocks, then, is not achieved. As monetary variability increases, the indexing parameter will increase, partially, but not fully, offsetting the increased monetary uncertainty. The partial nature of this offset stems from the increased cost of real shocks induced by rises in $\gamma$. The system is left, then, with a net increase in costs.

6. Concluding remarks

In this paper we have demonstrated the proposition that in an optimally indexed economy the real sector will not, in general, be fully insulated from the effects of monetary disturbances. This is because the optimal degree of indexing in such an economy is less than one if real shocks form part of the stochastic structure of the system. These conclusions rest on a distinction between price level fluctuations caused by monetary shocks and those caused by real shocks. The analysis implies that government policies which bring about increased monetary uncertainty impose unavoidable costs on the economy. Indexing cannot, in general, completely neutralize monetary variability; it appears, therefore, to be an inadequate substitute for intelligent behavior on the part of the monetary authority.

Appendix

1. Actual output

Actual output is given by eq. (1") of the text as a function of labor input, $L$, and the productivity factor, $\mu$. Actual labor input, in turn, is completely demand
determined and therefore given by eq. \((5'')\) of the text. Substituting \((5'')\) into \((1'')\) gives

\[
\log Y = -\delta \eta (\log w - \mu) + \mu + \delta \eta \log \delta. \tag{A1}
\]

But

\[
\log w = \log W - \log P
\]

\[
= \log W* + \gamma (\log P - \log P*) - \log P, \quad \gamma = \frac{\log W - \log W*}{\log P - \log P*}. \tag{A2}
\]

(Recall that asterisks denote certainty equivalents, or values corresponding to \(\mu = \xi = 0\).) Setting \(\log w* = 0\) results in \(\log W* = \log P*\), so that \((A2)\) can be written as

\[
\log w = (\gamma - 1)(\log P - \log P*). \tag{A3}
\]

Substituting \((A3)\) into \((A1)\) gives

\[
\log Y = -\delta \eta[(\gamma - 1)(\log P - \log P*) - \mu] + \mu + \delta \eta \log \delta. \tag{A4}
\]

Equating eqs. \((2'')\) and \((3'')\) of the text and solving for \(\log P\) yields

\[
\log P = \log \bar{M} + \xi - \log k - \log Y. \tag{A5}
\]

Setting \(\xi = \mu = 0\) gives

\[
\log P* = \log \bar{M} - \log k - \log Y*. \tag{A6}
\]

In turn, \(\log Y*\) is given by

\[
\log Y* = \delta \log L* = \delta \eta \log \delta. \tag{A7}
\]

(Recall that \(\log w* = 0\).) Substituting this result into \((A6)\) and subtracting \((A6)\) from \((A5)\) gives

\[
\log P - \log P* = \xi - \log Y + \delta \eta \log \delta. \tag{A8}
\]

Substituting \((A8)\) into \((A4)\) and solving for \(\log Y\) produces

\[
\log Y = \delta \eta \left[\frac{(1 - \gamma)\xi + \gamma \mu}{1 + \delta \eta(1 - \gamma)}\right] + \mu + \delta \eta \log \delta,
\]

which is \((12)\) of the text.
2. Desired output

Desired output is given by

\[ \log Y_o = \delta \log L_o + \mu, \quad \text{(A9)} \]

where desired labor input, \( L_o \), is determined by both supply and demand conditions in the labor market. Accordingly, \( \log L_o \) is found by equating eqs. (5') and (6') of the text, solving for \( \log w \), and substituting the resulting expression back into (6'):

\[ \log L_o = \mu(\omega \eta/(\omega + \eta)) + \eta \log \delta. \quad \text{(A10)} \]

Substitution of eq. (A10) into (A9) produces

\[ \log Y_o = \delta \eta \mu(\omega/(\omega + \eta)) + \mu + \eta \log \delta, \]

which is eq. (13) of the text.

References

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