8. Barro – Gordon – Model

Static Phillips curve: \( N = N_n + c (\pi - \pi^e) \),

2 goals of monetary policy:

1. Minimize deviations of inflation from its optimal rate \( \pi^* \)

2. Try to achieve efficient employment, \( N^* > N_n \)

Loss function \( L = b (\pi - \pi^*)^2 + (N - N^*)^2 \)

\[
\begin{align*}
\min L & \quad s.t. \quad N = N_n + c (\pi - \pi^e), \\
\min b(\pi - \pi^*)^2 + (N_n + c (\pi - \pi^e) - N^*)^2 \\
\Rightarrow \pi &= \frac{b \pi^* + c(N^* - N_n) + c^2 \pi^e}{b + c^2}
\end{align*}
\]

Response of the central bank on given expectations

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Barro – Gordon – Model

Rational expectations: \( \pi^e = E(\pi) \)

\[
\pi^e = E(\pi) = \frac{b \pi^* + c(N^* - N_n) + c^2 \pi^e}{b + c^2}
\]

\[
\Rightarrow \pi^e = \pi^* + \frac{c}{b}(N^* - N_n)
\]

\[
\Rightarrow \pi = \pi^* + \frac{c}{b}(N^* - N_n) \quad \text{discretionary solution}
\]

Inflation bias

Discretionary solution is inefficient!
Response of central bank to inflation expectations $\pi(\pi^e)$ without commitment

Equilibrium with rational expectations:

Discretionary solution = equilibrium

Phillips curve for $\pi^e = \pi^*$

Iso-loss curve
Commitment solution: central bank commits credibly to optimal inflation $\pi^*$.

Phillips curve for $\pi^e = \pi^*$

commitment solution

$\pi^*$

$N_n$

$N^*$

Iso-loss curve

Barro – Gordon – Model

How to avoid the inflation bias?
1. Reputation: Barro-Gordon (1983b)
2. Delegation: Rogoff (1985)
3. Central bank contract
4. Rules instead of discretionary decisions

Problem: trade-off between lowering inflation bias and optimal response to supply shocks.

Modern View of CBs: We do not try to achieve efficient employment, just to close the output gap.

i.e. $N^* = N_n \Rightarrow \pi = \pi^*$ in discretionary equilibrium
For avoiding the inflation bias and stabilizing the economy in an efficient way,

1. the CB must aim at closing the output gap, instead of trying to achieve efficiency, $N^* = N_n$,

2. the CB must be independent from short-run political goals of the government. Otherwise, there is always an incentive to increase employment in the short run, $N^* > N_n$.

With $L = b (\pi - \pi^*)^2 + (N - N_n)^2$ and $\pi^e = \pi^*$.

we get $L = b E(\pi - E(\pi))^2 + E(N - E(N))^2$

$= b \text{Var}(\pi) + \text{Var}(N)$.

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Stabilizing Demand and Supply Shocks

Stochastic economy:

- Liquidity shocks can be absorbed by monetary policy.
- Shocks of commodity demand: if CB stabilizes inflation, output and employment are stabilized as well.
- Supply shocks affect productivity. They are bound to have real effects.
8.1 Stabilizing supply shocks

What is the optimal response of monetary policy to shocks in productivity?

production function \( y = a N + \theta \)

Productivity shock \( \theta : \quad \mathbb{E}(\theta) = 0 \quad \text{Var}(\theta) = \sigma^2 \)

Labor demand \( N = N_n + c (\pi - w + \theta) \)

Wages \( w = \pi^e \)

All variables may be interpreted as growth rates.

Phillips curve

\[
N = N_n + c (\pi - \pi^e + \theta)
\]
Stabilizing supply shocks

Stabilizing inflation

\[ N = N_n + c (\pi - \pi^e + \theta) \]

Phillips curve for \( \theta = 0 \)

\[ \text{Var} (N) = c^2 \sigma^2 \]

employment fluctuations

Stabilizing employment

\[ \text{Var} (\pi) = \sigma^2 \]

fluctuations of inflation

Stabilizing supply shocks
Stabilizing supply shocks

Solving the trade-off between stabilizing inflation and employment.

\[
\text{Min } b (\pi - \pi^*)^2 + (N - N_n)^2 = \text{Min } b (\pi - \pi^*)^2 + (c (\pi^e + \theta))^2
\]

Since \( N^* = N_n \), the inflation bias is zero, so that \( \pi^e = \pi^* \).

\[\Rightarrow 2b (\pi - \pi^*) + 2c^2 (\pi - \pi^* + \theta) = 0 \]

\[\Rightarrow (b+c^2) (\pi - \pi^*) = -c^2 \theta \]

\[\Leftrightarrow \pi = \pi^* - \frac{c^2}{b+c^2} \theta \]
8.2 Rules versus Discretion

What are optimal rules in the face of demand and supply shocks?

production function \( y = aN + \theta \)

Productivity shock \( \theta : \ E(\theta) = 0 \quad \text{Var}(\theta) = \sigma_\theta^2 \)

Labor demand \( N = N_n + c(\pi - w + \theta) \), \( c = \frac{1}{1-a} \)

Wages \( w = \pi^e \)

Demand side (quantity theory) \( \mu + \eta = \pi + y \)

Demand shock \( \eta : \ E(\eta) = 0 \quad \text{Var}(\eta) = \sigma_\eta^2 \)

Loss function \( L = b(\pi - \pi^*)^2 + (N - N^*)^2 \)

---

Rules versus Discretion

Rule 1: constant rate of inflation, \( \pi = \pi^* \)

\[ \Rightarrow \pi^e = \pi^* \Rightarrow N = N_n + c \theta \]

\[
E[(N - N^*)^2 + b(\pi - \pi^*)^2] = E[(N_n - N^* + c\theta)^2]
\]

\[
= (N_n - N^*)^2 + 2(N_n - N^*)cE(\theta) + E[(c\theta)^2]
\]

\[
= (N_n - N^*)^2 + c^2 \sigma_\theta^2
\]
Rules versus Discretion

Rule 2: constant money growth rate, $\mu = \mu^* = \pi^* + aN_n$

\[ \pi = \mu + \eta - \gamma = \mu + \eta - aN - \theta = \pi^* + aN_n + \eta - aN - \theta \]

\[ E(\pi) = \pi^* + a(N_n - E(N)) + E(\eta) - E(\theta) = \pi^* \]

\[ E(N) = N_n \]

\[ N = N_n + c(\pi - \pi^e + \theta) \]
\[ = N_n + c((\pi^* + aN_n + \eta - aN - \theta) - \pi^* + \theta) \]
\[ = N_n + c(aN_n + \eta - aN) \]

\[ \iff \quad N = N_n + \eta \]

Rules versus Discretion

Rule 2: constant money growth rate

Inflation

\[ \pi = \pi^* + a(N_n - N) + \eta - \theta = \pi^* - \alpha \eta + \eta - \theta = \pi^* + (1 - \alpha) \eta - \theta \]

Expected welfare loss

\[ E[(N - N^*)^2 + b(\pi - \pi^*)^2] \]
\[ = E[(N_n - N^* + \eta)^2 + b((1 - \alpha) \eta - \theta)^2] \]
\[ = (N_n - N^*)^2 + 2(N_n - N^*)E(\eta) + E(\eta^2) \]
\[ + b(1 - \alpha)^2 E(\eta^2) - 2b(1 - \alpha)E(\eta \theta) + bE(\theta)^2 \]
\[ = (N_n - N^*)^2 + \sigma^2 + b(1 - \alpha)^2 \sigma^2 + b \sigma^2 \]
Rules versus Discretion

Comparing welfare loss for rule 1 versus rule 2

\[ E(\text{loss for constant } \mu) > E(\text{loss for constant } \pi) \]
\[ \Leftrightarrow \quad (N_n - N^*)^2 + \sigma^2 + b(1-\alpha)^2 \sigma^2 + b \sigma^2 > (N_n - N^*)^2 + \frac{\sigma^2}{(1-\alpha)^2} \]
\[ \Leftrightarrow \quad (1 + b(1-\alpha)^2) \sigma^2 > \left( \frac{1}{(1-\alpha)^2} - b \right) \sigma^2 \]
\[ \Leftrightarrow \quad \frac{\sigma^2}{\sigma^2} > \frac{1}{(1-\alpha)^2} - b \quad \Leftrightarrow \quad b > \frac{1}{(1-\alpha)^2} \]

Constant money supply growth leads to higher expected costs than constant inflation, if (i) the weight on price stability is sufficiently large, or (ii) the variance of demand shocks is large compared to the variance of supply shocks.